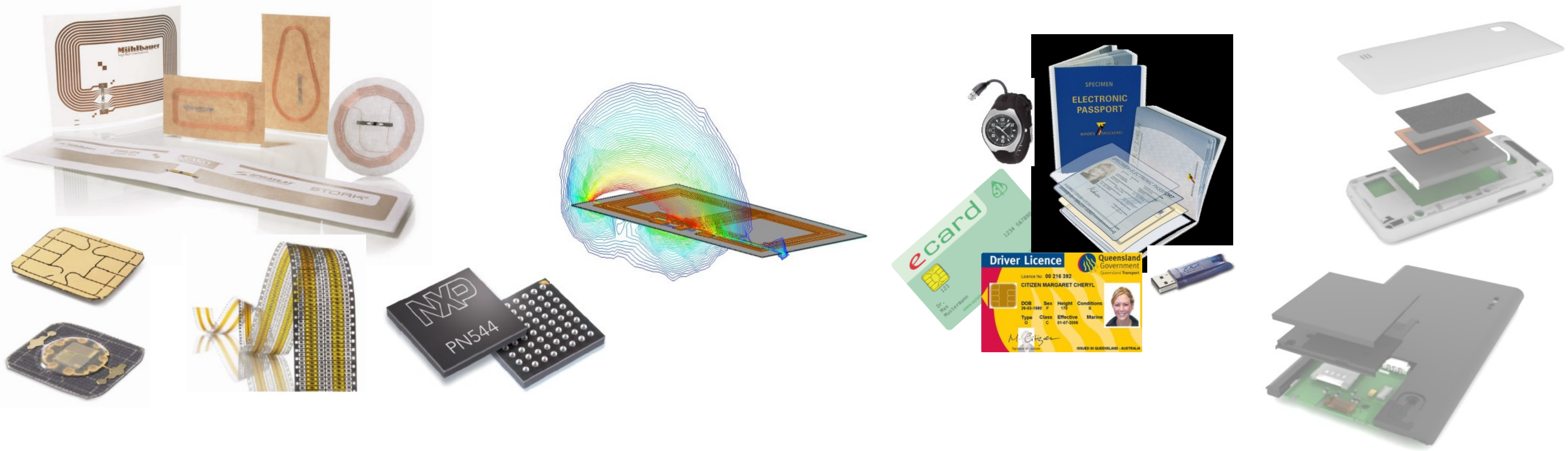

03 HF Basics

3rd unit in course 451.417, RFID Systems, TU Graz

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Advancing Technology

Content

Fields

Overview

Contactless power transmission in near-field

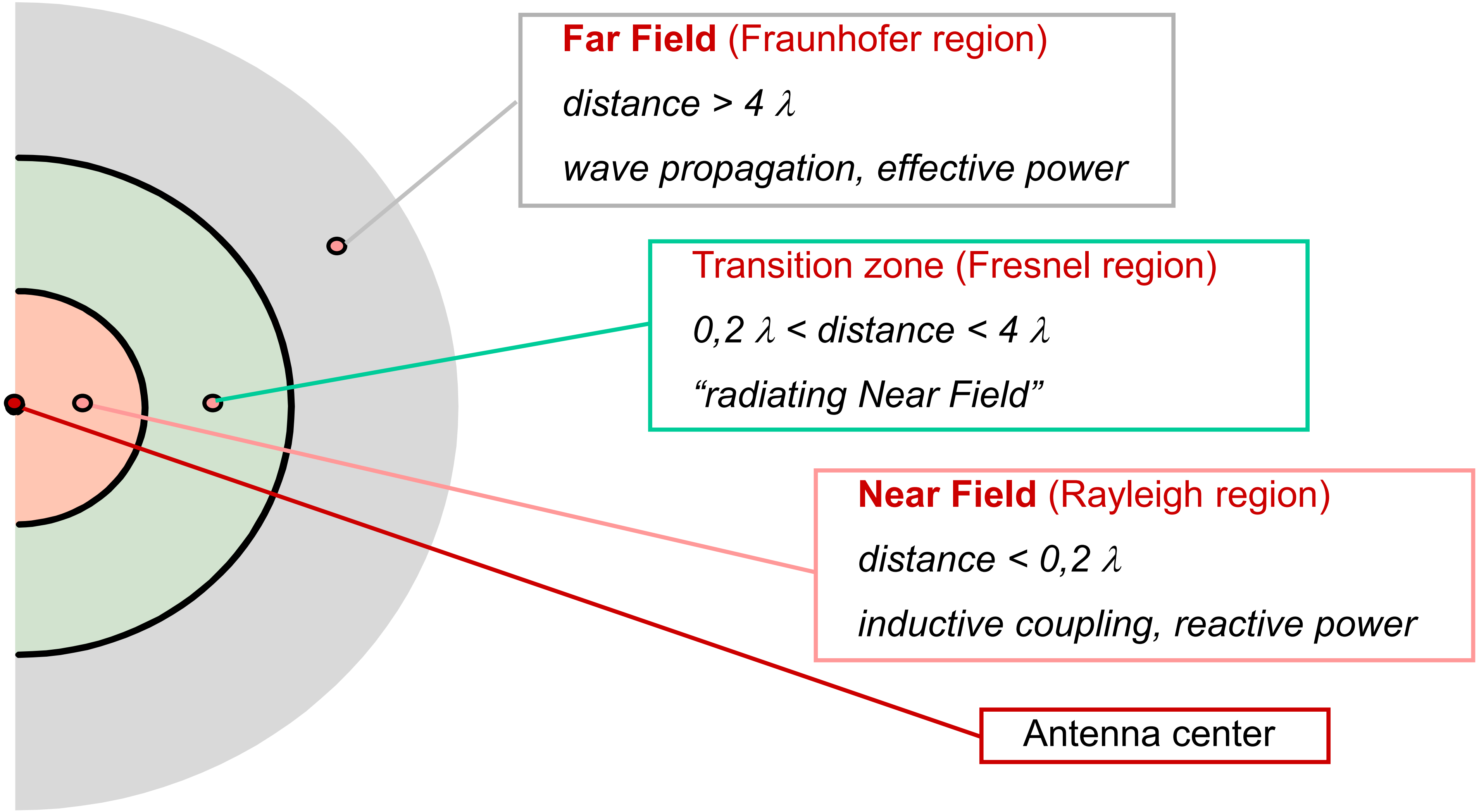
- Contactless power transmission almost exclusively works over the alternating *H*-field, in inductively coupled systems in the near-field. This is surrounding each current-carrying conductor and induces voltages in conductors near by.
- In the proximity of a loop antenna, free propagation of an electromagnetic wave is not yet given, and the *E*-field is very weak, compared to the *H*-field. Moreover, there is a phase-shift (of almost 90 °) between *E* and *H*, so the wave impedance is complex (almost imaginary) and the EM wave carries reactive energy.
- The *H*-field strength decreases by $1/d^3$ (- 60 dB/Dec.) in the near field, while the decay in far field is $1/d^1$ (- 20 dB/Dec.) for *H*-field as well as for *E*-field.
- As the Emission Limits for allowed *H*-field radiation are normally measured in a constant distance (EU: 10 m, US: 30 m), which already is in the far-field for 13,56 MHz, using the *H*-field at HF frequencies has the benefit to allow quite high power transmission over short distances from Reader to Transponder (which is a few cm for person-related card systems).

Concepts to estimate the emitted *H*-field

- **Magnetic Momentum**
  - This method was developed by Heinrich Hertz in analogy to the dipole momentum for the calculation of the *E*-field. It delivers good results for the far-field or in sufficient distance to the emitting conductor. Conductor geometry is not considered (circular equivalent).

- **Biot-Savart Law**
  - This method takes the geometry of the current-carrying conductor into account and delivers accurate results for the *H*-field emission in the near field. The original equation does not take wave propagation into account (and fails in the far field).
  - The formula can be extended by a “retardation potential” such that wave propagation is also taken into account. So it delivers good results in near– and far-field.

Near Field, transition zone, Far Field

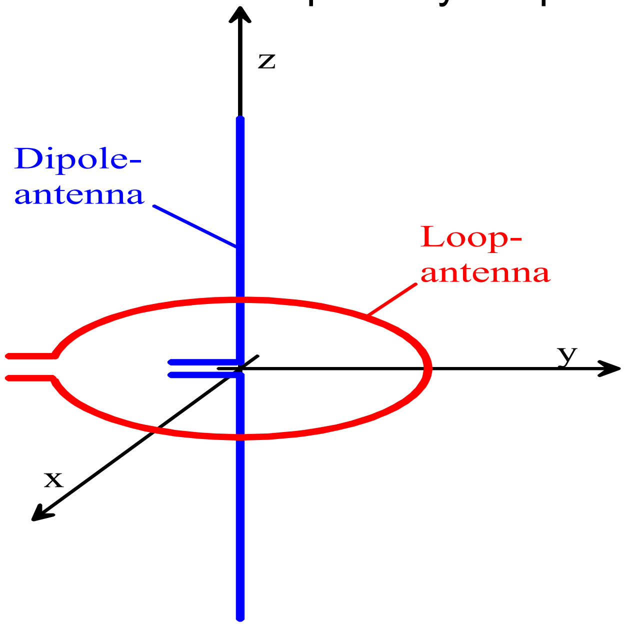


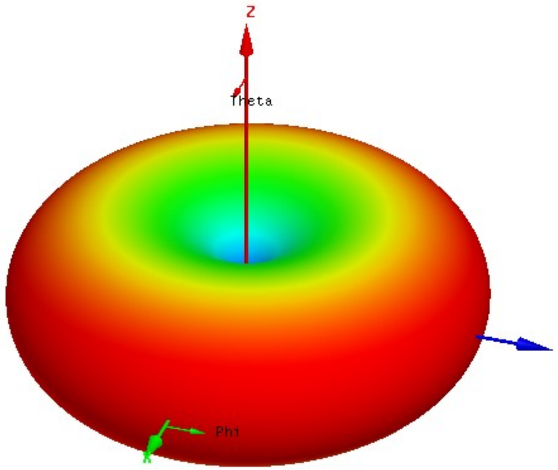
The Magnetic Momentum method

Estimating the emitted *H*-field independent of antenna geometry (far field)

The magnetic momentum

- Heinrich Hertz developed the method of the magnetic momentum to calculate the *H*-field strength in space, in analog to the electric dipole momentum.
- It delivers good results in the far field of an antenna
- The magnetic momentum for a conductor loop of any shape is given by the alternating current times the area inside the loop $\left|\vec{m}_d\right| = I \cdot \int d\vec{A}$





The magnetic momentum

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- It delivers good results in the far field of an antenna
- The magnetic momentum for a conductor loop of any shape is given by the alternating current times the area inside the loop $|\vec{m}_d| = I \cdot \int d\vec{A}$

  - More general, for arbitrary current distribution in the space volume:

$$|\vec{m}_d| = \frac{1}{2} \cdot \int \vec{r} \times \vec{J} dV$$

$dV = \left(r^2 \sin\theta\right) dr\, d\theta\, d\Phi$ …space volume element

$\vec{r}$ …direction from origin to space element

$\vec{J} = \rho\, \vec{v}$ …current density times velocity

  - In practice we find for the absolute of the momentum for a planar loop:
  - for a circular loop:

$$|\vec{m}_d| = N \cdot I \cdot A = N\left(r^2 \pi\right) \cdot I$$

  - for a rectangular loop:

$$|\vec{m}_d| = N \cdot I \cdot A = N\left(l \cdot b\right) \cdot I$$

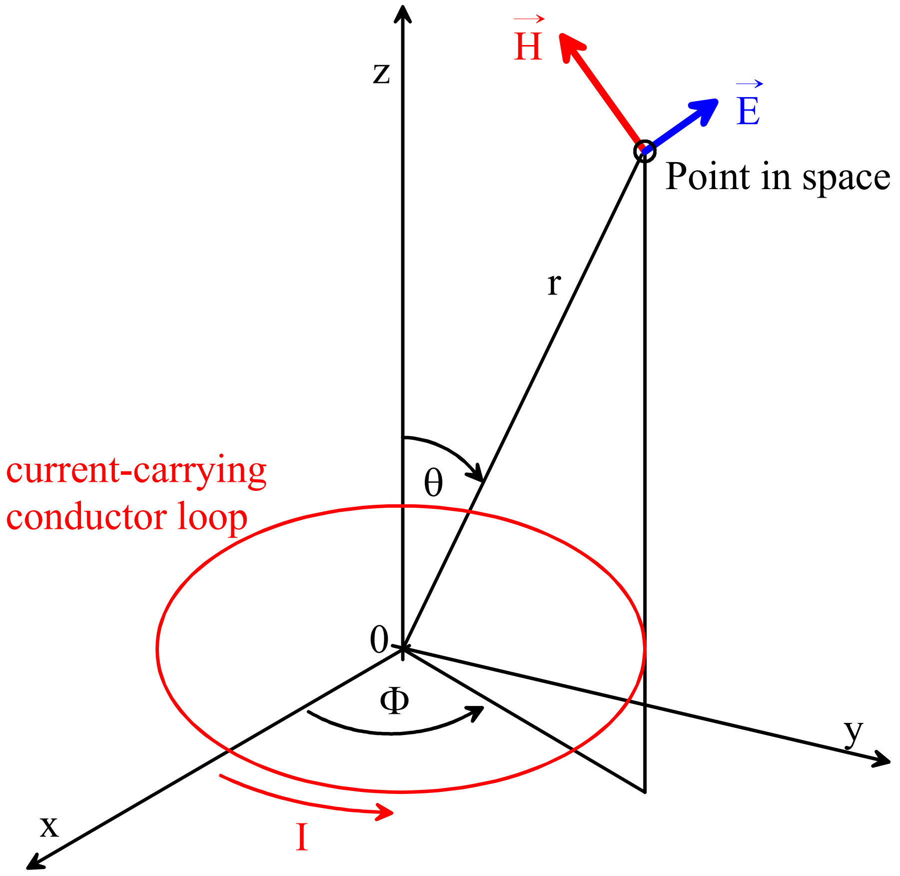
H-field expressed by the magnetic momentum

- The *H*-field and the *E*-field (of a current-carrying conductor loop) in space can be derived from the magnetic momentum for any point in space in spherical coordinates by

$$\vec{H} = \frac{\vec{m}_d}{4\pi ƛ^2 r}\left\{2\left(\frac{ƛ^2}{r^2} + j\frac{ƛ}{r}\right)\cos\theta\,\hat{\Phi} + \left(-1 + \frac{ƛ^2}{r^2} + j\frac{ƛ}{r}\right)\sin\theta\hat{\theta}\right\}$$

$$\vec{E} = \frac{\mu_0 c \cdot \vec{m}_d}{4\pi ƛ^2 r}\left(1 - j\frac{ƛ}{r}\right)\sin\theta\,\hat{\Phi}$$

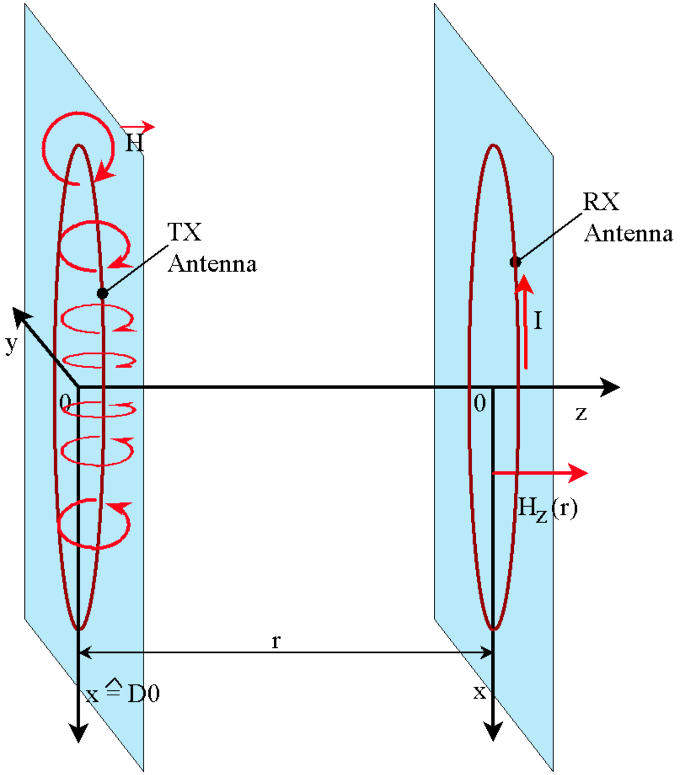
$$ƛ = \frac{\lambda}{2\pi}$$ …radian wavelength, angular wavelength

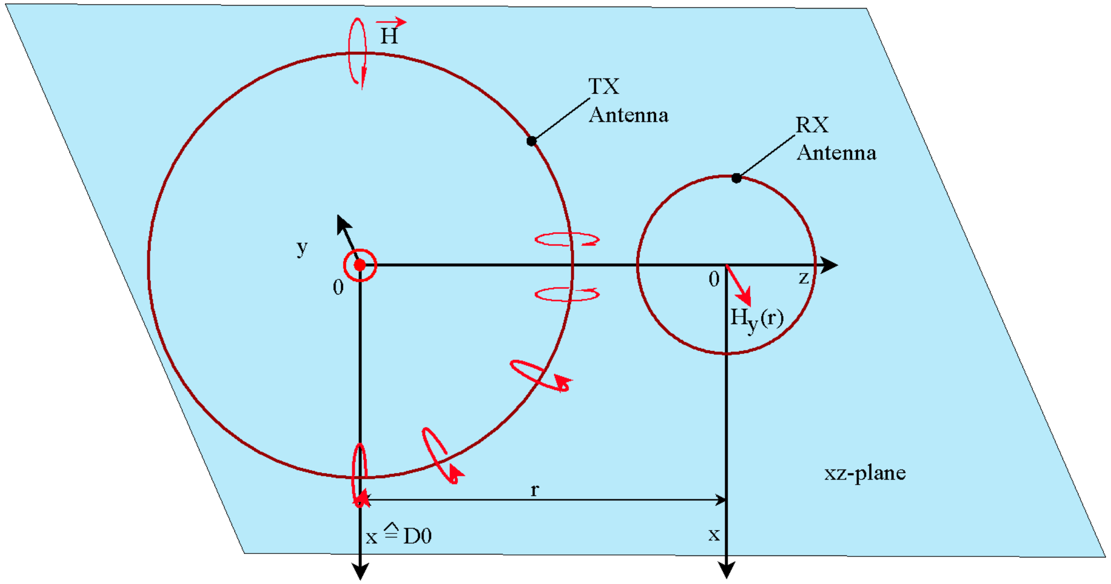


- The relation of *E*-field to *H*-field gives the field impedance *Z*

$$Z = \frac{E}{H}$$

Dedicated Antenna Orientations





Coaxial orientation

- Center points of both antenna conductors are on an axis perpendicular to the antenna plane

Coplanar orientation

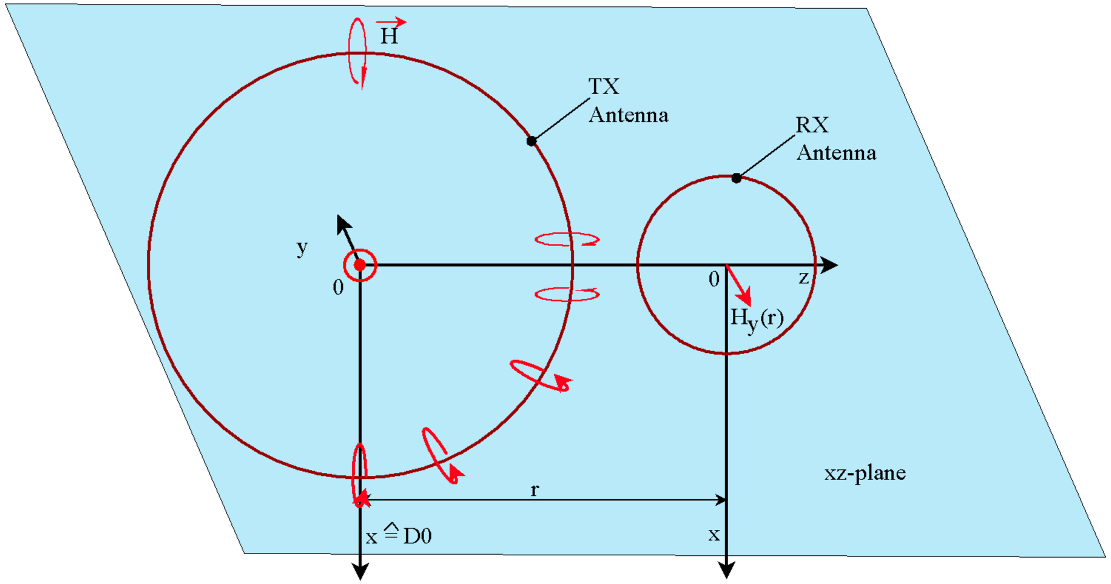
- both antenna conductors are in the same plane

H-field and *E*-field for Coplanar Orientation

- **Coplanar orientation**
  - the angle to the antenna axis is 90°

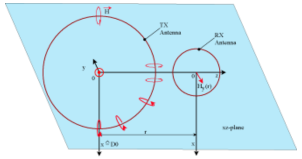
$$\theta \equiv 90 \quad \Rightarrow \quad \cos(\theta) = 0$$

$$\Rightarrow \quad \sin(\theta) = 1$$



¬ the cosine term disappears and a simplified equation for *H*- and *E*-field remains

$$\vec{H} = \frac{\vec{m}_d}{4\pi\,\lambda^2 r}\left(-1 + \frac{\lambda^2}{r^2} + j\frac{\lambda}{r}\right) \qquad \vec{E} = \frac{\mu_0 c \cdot \vec{m}_d}{4\pi\,\lambda^2 r}\left(1 - j\frac{\lambda}{r}\right)$$



Field Impedance Z for Coplanar Orientation

– As *E*-field and *H*-field are non zero, the field impedance *Z* can be calculated from the magnitude of *E* and *H*:

$$\left|\vec{H}\right| = \frac{\left|\vec{m}_d\right|}{4\pi\,\bar{\lambda}^2 r}\sqrt{\left(-1+\frac{\bar{\lambda}^2}{r^2}\right)^2+\left(\frac{\bar{\lambda}}{r}\right)^2} = \frac{m_d}{4\pi\,\bar{\lambda}^2 r}\,\frac{1}{r^2}\sqrt{r^4 - r^2\bar{\lambda}^2 + \bar{\lambda}^4}$$

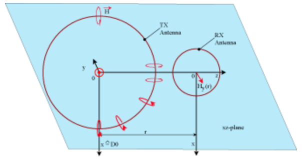
$$\left|\vec{E}\right| = \frac{\mu_0 c\cdot\left|\vec{m}_d\right|}{4\pi\,\bar{\lambda}^2 r}\sqrt{(1)^2+\left(-\frac{\bar{\lambda}}{r}\right)^2} = \frac{\mu_0 c\cdot m_d}{4\pi\,\bar{\lambda}^2 r}\,\frac{1}{r}\sqrt{r^2+\bar{\lambda}^2}$$

$$Z = \frac{E}{H} = $$

$$= \frac{\dfrac{\mu_0 c\cdot m_d}{4\pi\,\bar{\lambda}^2 r}\left(1-j\dfrac{\bar{\lambda}}{r}\right)}{\dfrac{m_d}{4\pi\,\bar{\lambda}^2 r}\left(-1+\dfrac{\bar{\lambda}^2}{r^2}+j\dfrac{\bar{\lambda}}{r}\right)} =$$

$$= Z_0\cdot\frac{\left(1-j\dfrac{\bar{\lambda}}{r}\right)}{\left(-1+\dfrac{\bar{\lambda}^2}{r^2}+j\dfrac{\bar{\lambda}}{r}\right)} = \ldots =$$

$$= Z_0\,\frac{r^4 + jr\bar{\lambda}^3}{\left(\bar{\lambda}^4 - \bar{\lambda}^2 r^2 + r^4\right)}$$



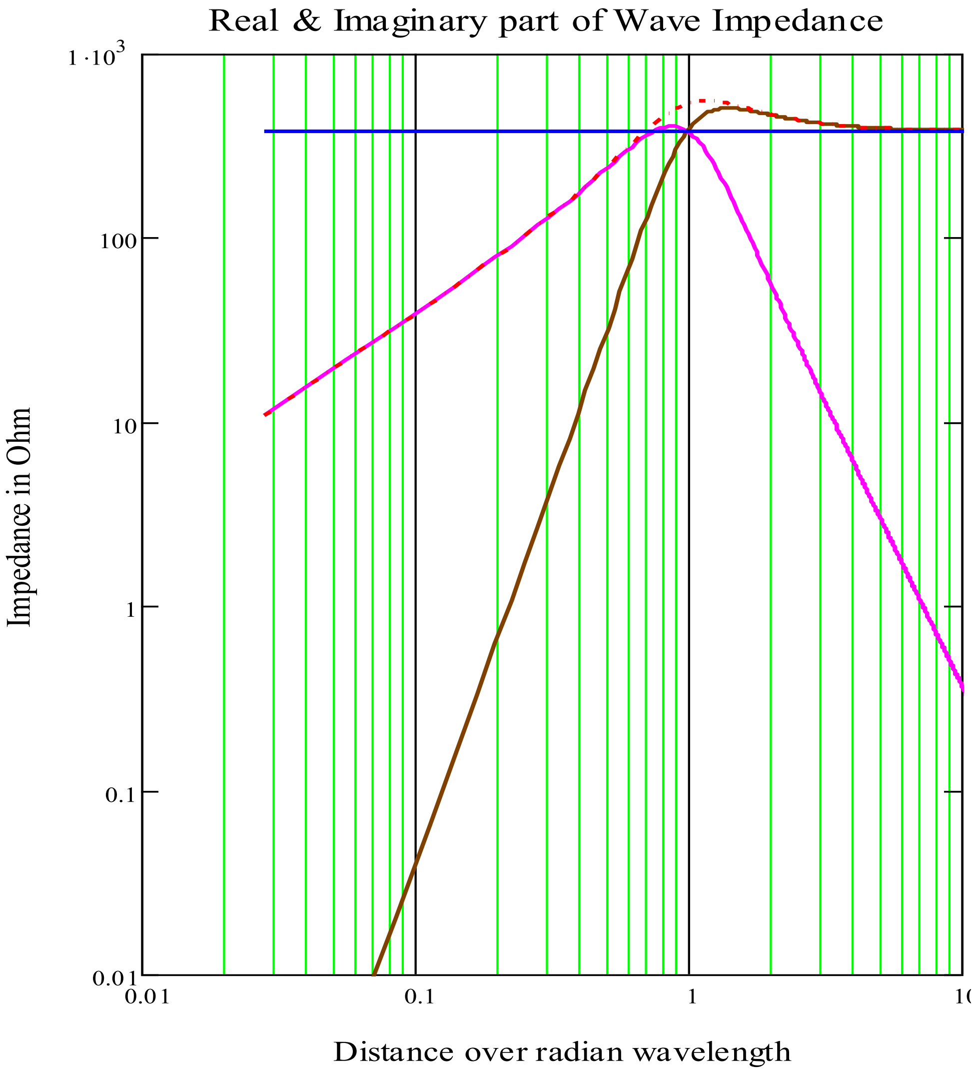
Field Impedance *Z* for Coplanar Orientation

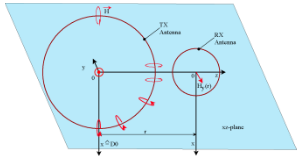
– We can separate the real part and the imaginary part of *Z*:

$$\mathrm{Re}(Z) = Z_0 \frac{r^4}{\left(\bar{\lambda}^4 - \bar{\lambda}^2 r^2 + r^4\right)}$$

$$\mathrm{Im}(Z) = Z_0 \frac{r\bar{\lambda}^3}{\left(\bar{\lambda}^4 - \bar{\lambda}^2 r^2 + r^4\right)}$$

$$Z_0 = \frac{E}{H} = \frac{\dfrac{\mu_0 c \cdot m_d}{4\pi\,\bar{\lambda}^2 r}}{\dfrac{m_d}{4\pi\,\bar{\lambda}^2 r}} = \mu_0\, c$$





Field Impedance Z for Coplanar Orientation

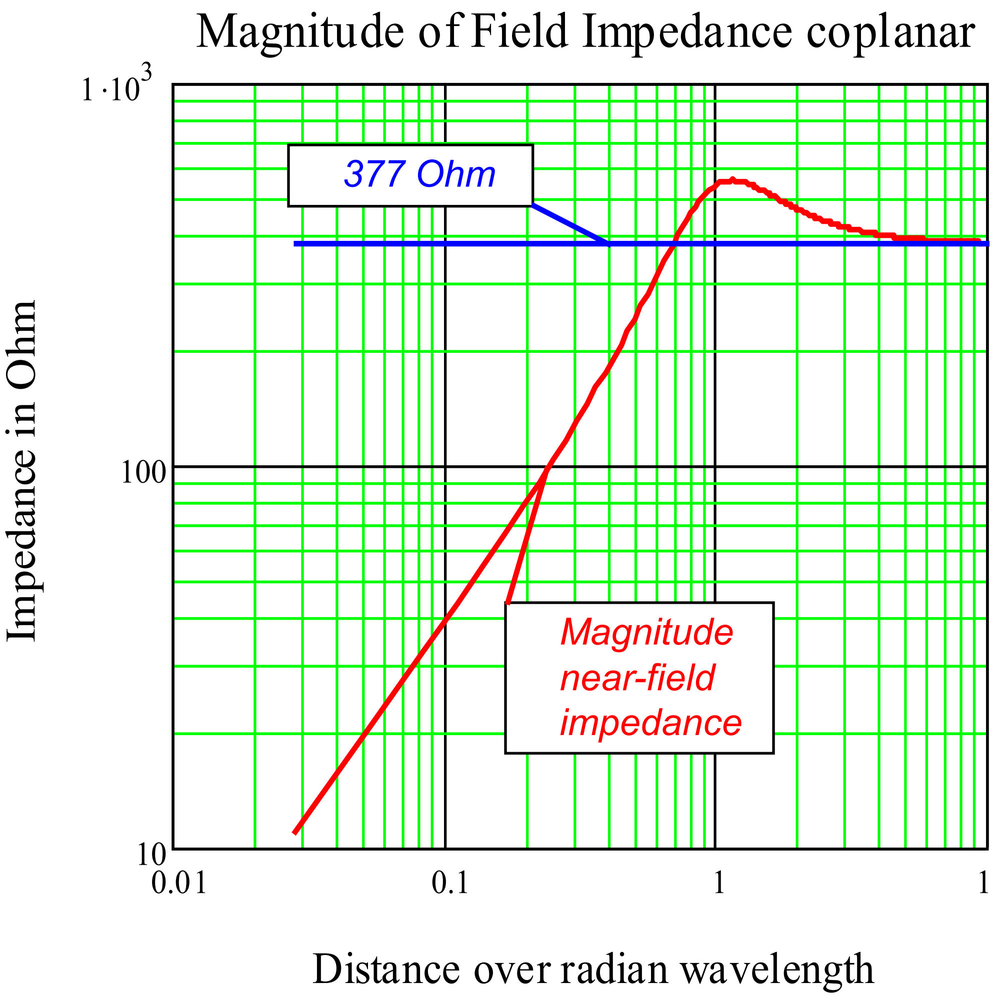
- The magnitude of *Z* can be calculated from real and imaginary part:

$$|Z| = \sqrt{\mathrm{Re}^2(Z) + \mathrm{Im}^2(Z)} = \frac{|E|}{|H|} =$$

$$= Z_0 \frac{r\sqrt{r^6 + \bar{\lambda}^6}}{\left(\bar{\lambda}^4 - \bar{\lambda}^2 r^2 + r^4\right)}$$

- In the far field the field impedance *Z* approximates the wave impedance Z_0

$$Z_0 = \mu_0 \cdot c = \frac{1}{\varepsilon \cdot c} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega \cong 377\ \Omega$$



H-field and *E*-field for Coaxial Orientation

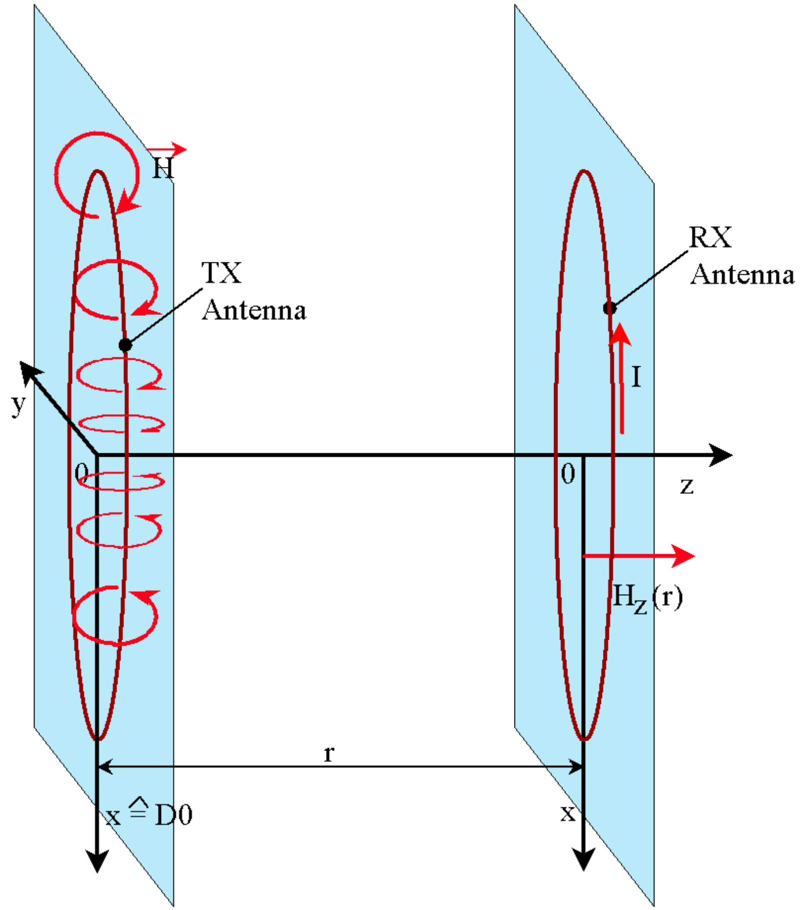
- **Coaxial orientation**
  - the angle to the antenna axis is zero

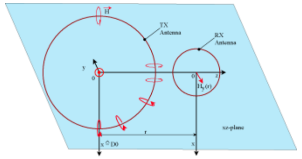
$$\theta \equiv 0 \quad \Rightarrow \quad \cos(\theta) = 1$$
$$\Rightarrow \quad \sin(\theta) = 0$$

  - the *E*-field component disappears and only the *H*-field component remains

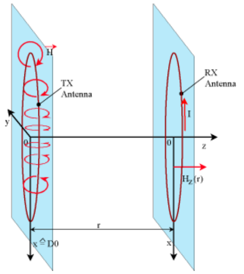
$$\vec{E}(\theta = 0) \equiv 0 \qquad \vec{H}(\theta = 0) = \frac{2\vec{m}_d}{4\pi \, ƛ^2 r}\left(\frac{ƛ^2}{r^2} + j\frac{ƛ}{r}\right)$$

  - consequently the field impedance Z becomes zero for coaxial orientation!





H-field comparison for coplanar and coaxial orientation



- **Coplanar orientation**
  - complex field vector

$$\vec{H}(\theta=90^\circ)=\frac{\vec{m}_d}{4\pi\,\lambda^2 r}\left(-1+\frac{\lambda^2}{r^2}+j\frac{\lambda}{r}\right)$$

  - magnitude

$$\left|\vec{H}\right|=\frac{\left|\vec{m}_d\right|}{4\pi\,\lambda^2 r}\sqrt{\left(-1+\frac{\lambda^2}{r^2}\right)^2+\left(\frac{\lambda}{r}\right)^2}=$$

$$=\frac{m_d}{4\pi\,\lambda^2 r}\frac{1}{r^2}\sqrt{r^4-r^2\lambda^2+\lambda^4}$$

- **Coaxial orientation**
  - complex field vector

$$\vec{H}(\theta=0^\circ)=\frac{2\vec{m}_d}{4\pi\,\lambda^2 r}\left(\frac{\lambda^2}{r^2}+j\frac{\lambda}{r}\right)$$

  - magnitude

$$\left|\vec{H}\right|=\sqrt{\mathrm{Re}^2+\mathrm{Im}^2}=$$

$$=\frac{2m_d}{4\pi\,\lambda^2 r}\sqrt{\left(\frac{\lambda^2}{r^2}\right)^2+\left(-\frac{\lambda}{r}\right)^2}=\ldots=$$

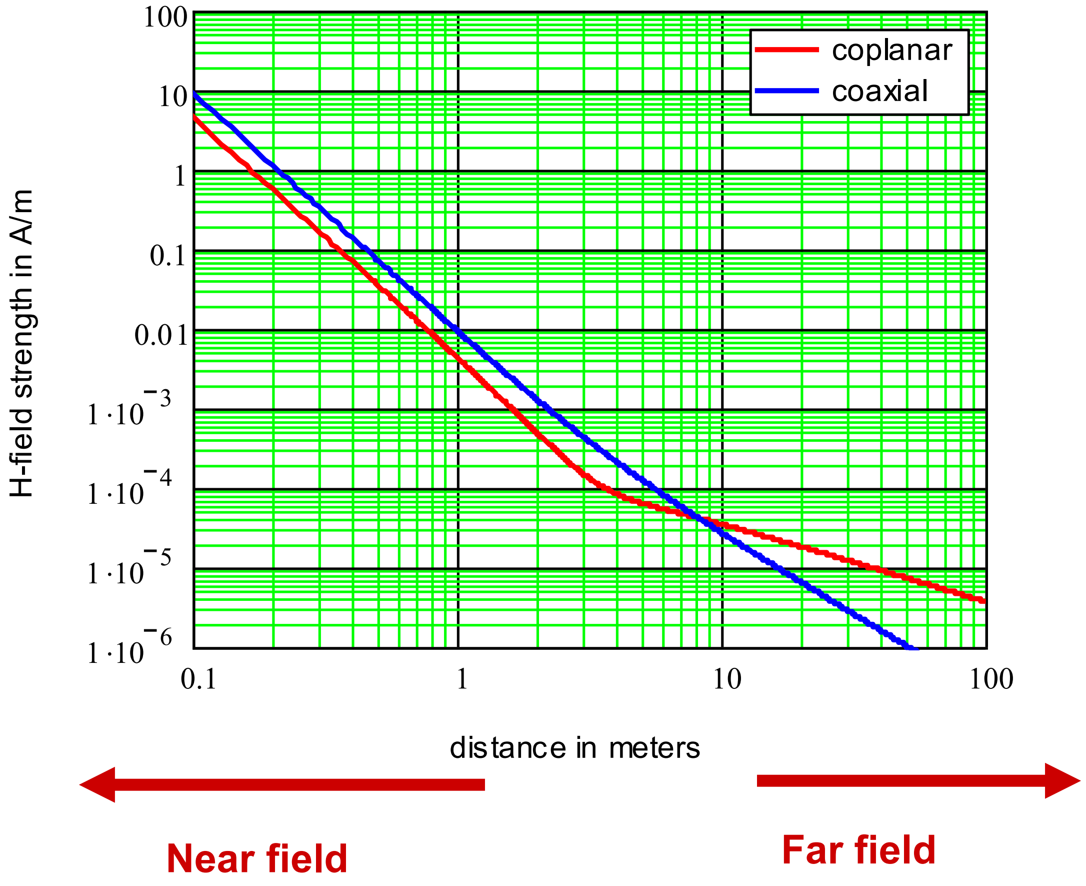
$$=\frac{m_d}{2\pi\,\lambda^2}\frac{1}{r^3}\sqrt{\lambda^4+\lambda^2 r^2}$$

H-field comparison for coplanar and coaxial orientation

Decrease of the *H*-field with distance for coaxial and coplanar antenna orientation:

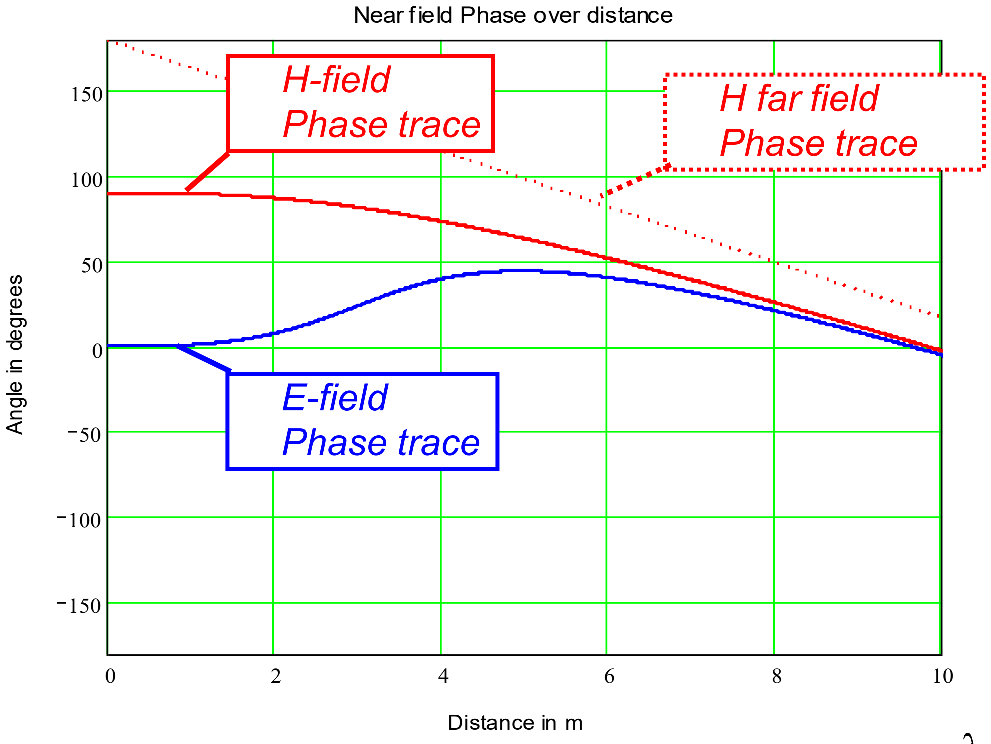
- **Near field**
  - Coaxial & coplanar: $H \sim 1/d^3$
- **Far field**
  - Coaxial: $H \sim 1/d^3$
  - Coplanar: $H \sim 1/d^1$

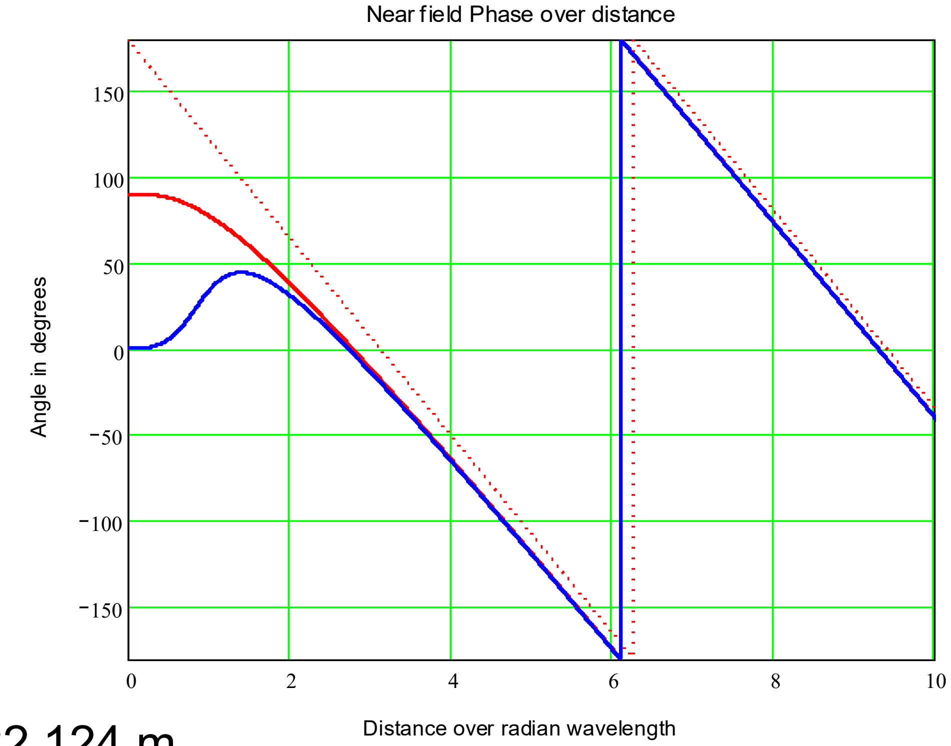
λ at 13,56 MHz = 22,124 m



Phase trace in the Near Field

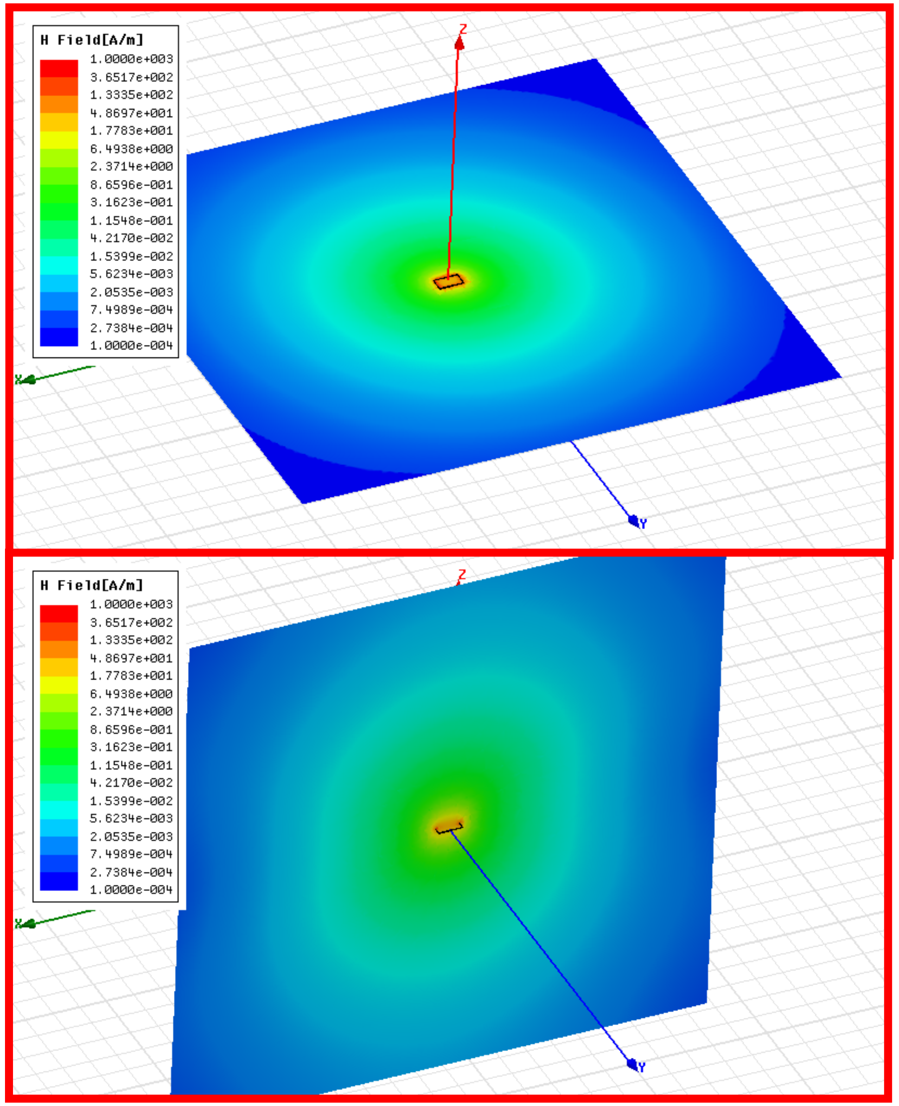
- **Hertz** also investigated the phase trace over distance. **John Wheeler** defined the "radiation sphere" $r = \lambda / 2\pi$ to define the near-field zone.
  - In the near field close to the conductor, the field is directly linked to current.
  - Wave propagation in the far field follows the relation $c = \lambda f$.

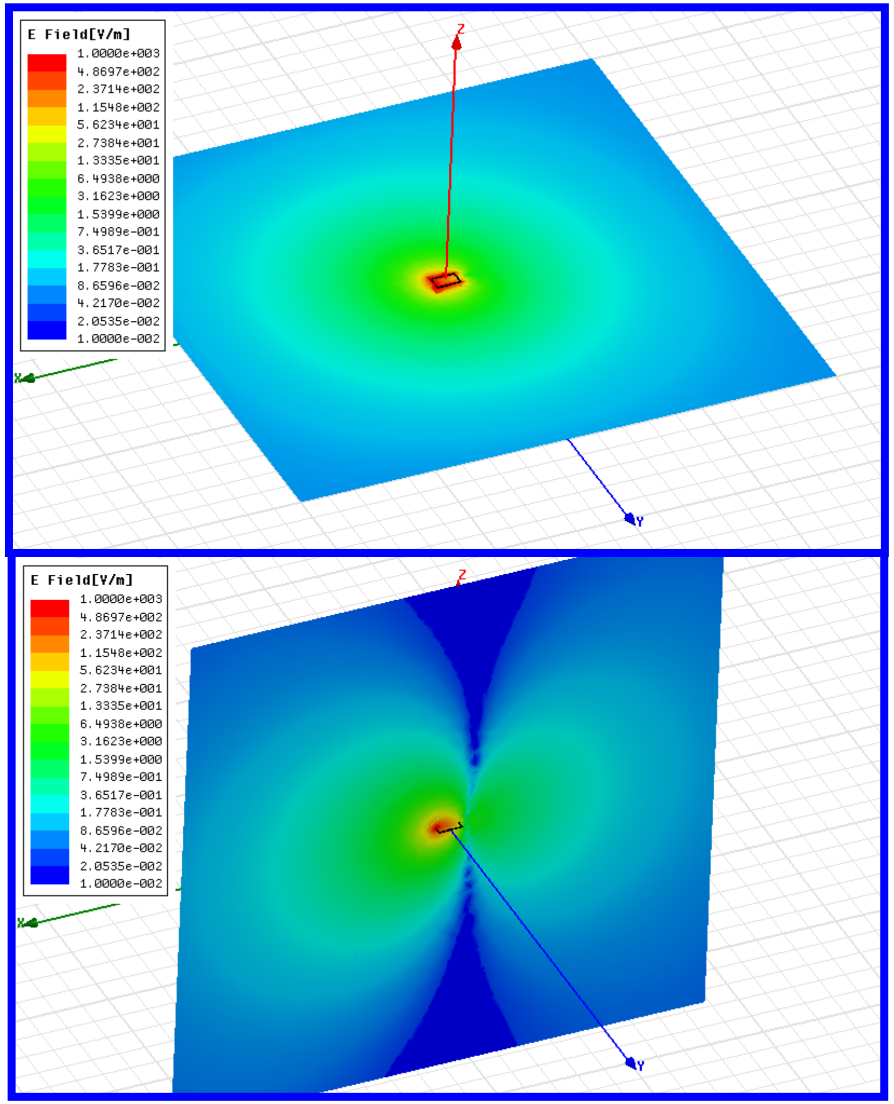




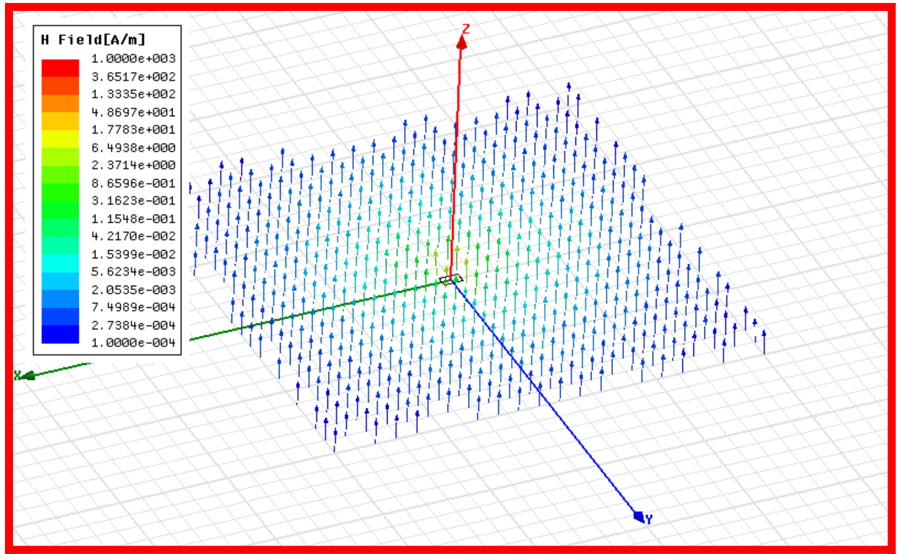
λ at 13,56 MHz = 22,124 m

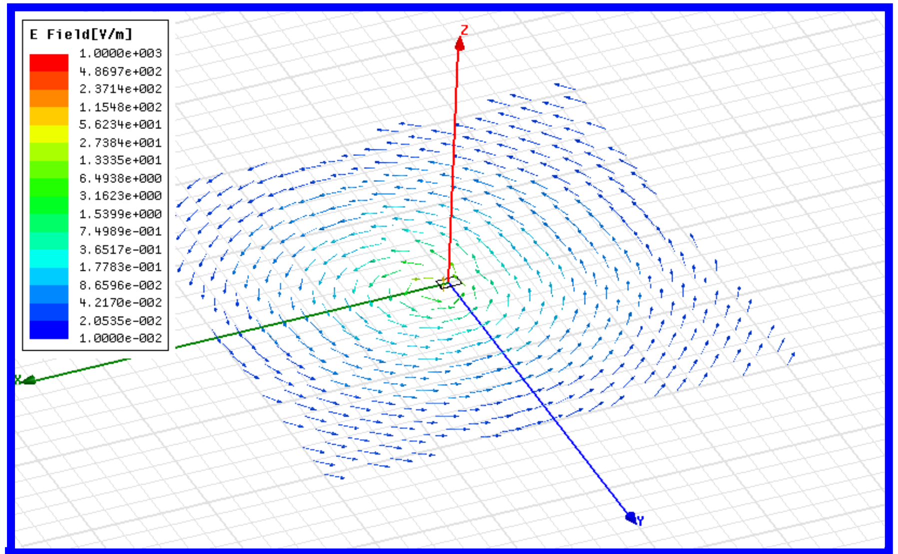
H-field and *E*-field magnitude

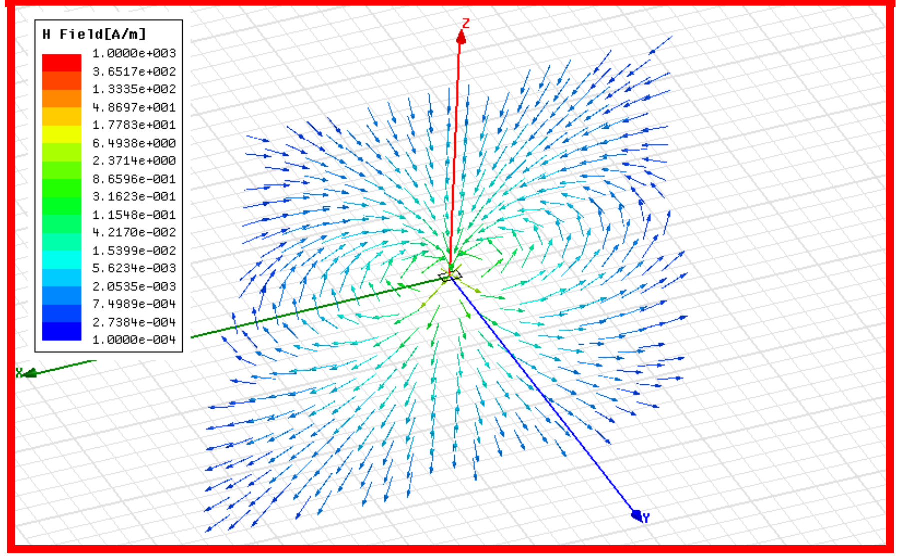


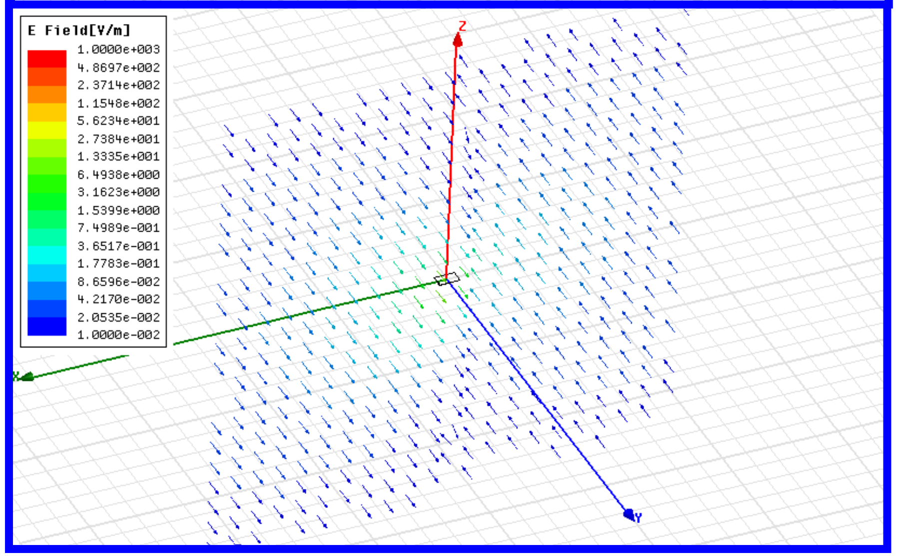


H-field and *E*-field direction









Absolute and decibel values

- Decibel values are logarithmic, relative to an absolute reference value.
  - Power scales 10 times the logarithm, *H*-field and *E*-field (like current and voltage) scale 20 times the logarithm, as they are square-proportional to power.

$$H_{dB} = 20\log\left(\frac{H_{ABS}}{H_{REF}}\right) \qquad H_{ABS} = H_{REF} \cdot 10^{H_{dB}/20}$$

$$Z_{0,dB} \cong 20\log(377\ \Omega) \cong 51.5\ dB_{(\Omega)}$$

e.g. *H*-field emission limit 60 dB(µA/m) is in absolute units…

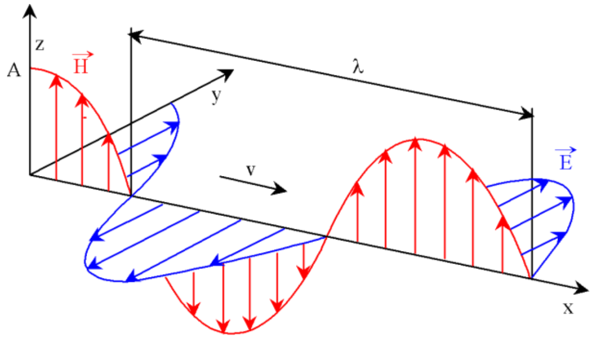
$$H_{LIMIT} = H_{REF} \cdot 10^{H_{dB}/20} = 1\mu A/m \cdot 10^{60dB/20} = 1000\ \mu A/m = 1mA/m$$

e.g. for far field the related magnitude of the *E*-field is …

$$E_{LIMIT} = H_{LIMIT} \cdot Z_0 = 1000\ \mu A/m \cdot 377\ \Omega = 377000\ \mu V/m$$

Same calculation in decibel values…

$$E_{dB} = H_{dB} + Z_{0,dB} = H_{dB} + 51.5 dB_{(\Omega)} = 60\, dB(\mu A/m) + 51.5\, dB(\Omega) = 111.5\, dB(\mu V/m)$$



electromagnetic wave in the far field

"Wireless" power transmission

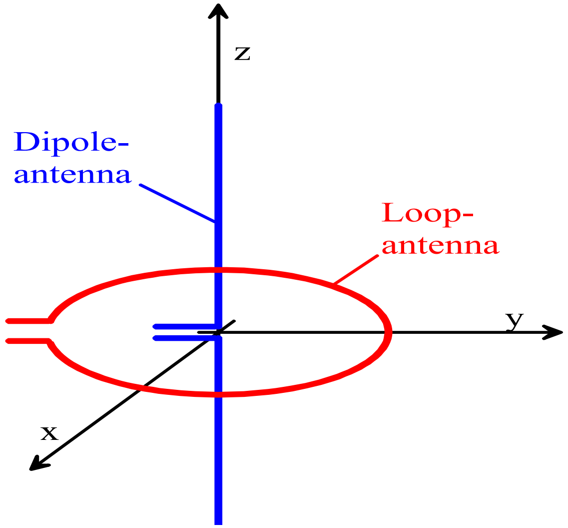
– Power density can be derived with the Poynting vector concept $\vec{S} = \vec{E} \times \vec{H}$

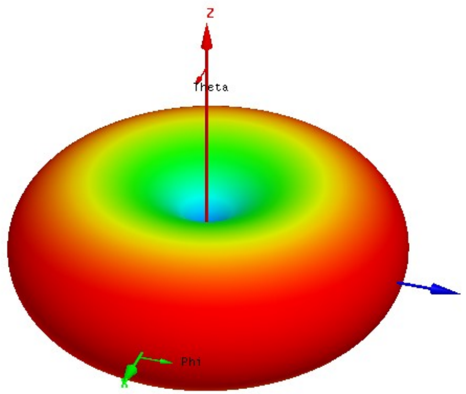
– for field magnitudes in the far field this means $S = \frac{E^2}{Z_0} = Z_0 \cdot H^2$

$$P = \int_A \vec{S} \circ d\vec{A} = A \frac{E^2}{Z_0} = A Z_0 \cdot H^2$$

– antenna directivity *D* is $D = \frac{S}{S_{ISOTROPH}}$

– For a loss-less antenna, directivity is equal to gain





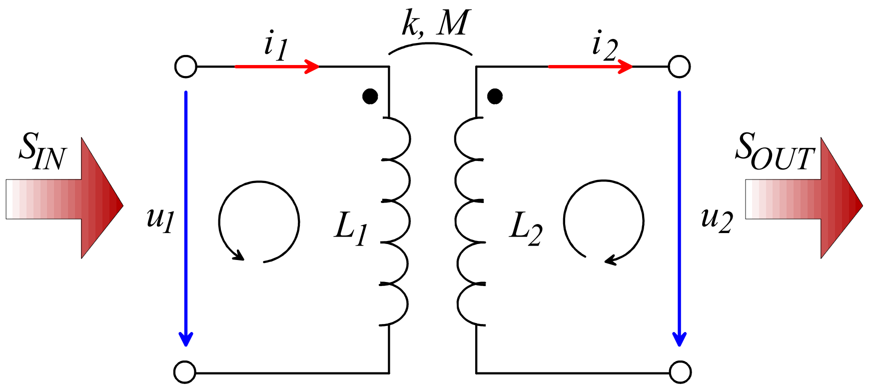
Gain of a planar loop in the far field

– e.g. an *H*-field emission limit of 60 dB(µA/m) in far field is related to a power density of…

$$S = Z_0 \cdot H^2 = 377\,\Omega \cdot (1mA/m)^2 = 377 \cdot 10^{-4}\,W/m^2 = 0{,}377\ mW/m^2$$

"Contactless" power transmission

- An ideal transformer is a good model for contactless power transmission
  - we neglect losses, resonances and inductances are linear and time-invariant



- coil flux $\psi(t) = \phi(t)\cdot n = B(t)A\,n$

- branches $\psi_1(t) = L_{11}i_1(t) - L_{12}i_2(t)$

$$\psi_2(t) = L_{22}I_2(t) - L_{21}I_1(t),$$

- network equations

$$U_1 = j\omega L_1 I_1 - j\omega M\, I_2$$

$$U_2 = j\omega L_2 I_2 - j\omega M\, I_1.$$

  - using harmonic sine wave signals… $\frac{d}{dt} \to j\omega$
  - …where *U* and *I* are root-mean-square (rms) values of the signals *u(t)* and *i(t)*

$$S_{IN} = U_1 I_1 = \left[j\omega L_1 + \frac{\omega^2 M^2}{Z_L - j\omega L_2} \right] I_1^{\,2},$$

  - the apparent power S of the primary circuit is given by…
    (without secondary load current, this is just reactive)

$$S_{OUT} = I_2^{\,2} Z_L = \left(-\frac{j\omega M}{Z_L - j\omega L_2} \right)^2 I_1^{\,2} Z_L.$$

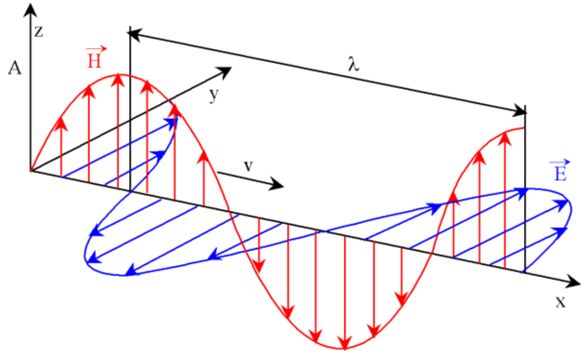
  - for effective power transmission, we need to consider…

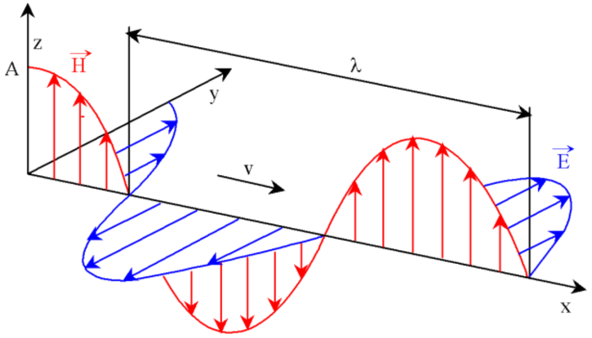
$$P = \mathrm{Re}[S]$$

Summary Near Field, Far Field

Near field | **Far field**

- Electromagnetic wave





- Limit of field region

  Near field: $$\frac{\lambda}{2\,\pi} \geq 3 \cdot \text{distance}$$

  Far field: $$\frac{\lambda}{2\,\pi} \leq 0{,}3 \cdot \text{distance}$$

- Antenna diameter D_0

  Near field: $$\text{distance} \geq 2\frac{D_0^{\,2}}{\lambda}$$

  Far field: $$\text{distance} \leq 2\frac{D_0^{\,2}}{\lambda}$$

- Phase-shift in time

  Near field: $$\angle\big(E(t), H(t)\big) \neq 0°$$

  Far field: $$\angle\big(E(t), H(t)\big) = 0°$$

- Spatial field vectors

  $$\vec{E} \perp \vec{H}$$

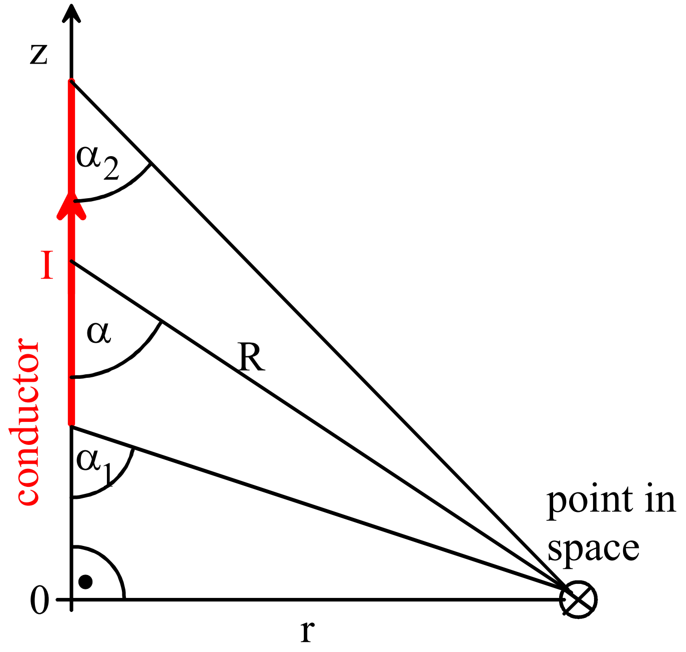
Biot-Savart Law

Calculating emitted alternating *H*-field close to the conductor

Law of Ampére

- As a first step, Jean-Marie Ampére found a rule in 1820. Current in a conductor generates an *H*-field around the conductor. Using todays notation of units, his rule is…

$$B_\Phi = \frac{\mu_0 I}{4\pi r} \cdot (\cos\alpha_2 - \cos\alpha_1)$$



I…….antenna current

r……distance from conductor center

μ_0….permeability (magnetic field constant),

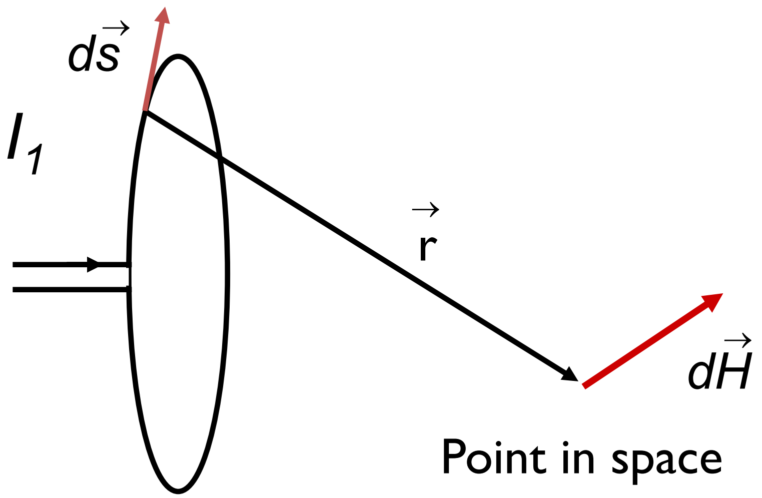
in free space $4\pi\ 10^{-7}$ Vs / Am

  – for the special case of an infinitely long, straight conductor (➔ a_1 = -180 °, a_2 = 0 °) results the well-known simple relation…

$$B_\Phi = \frac{\mu_0 I}{2\pi r} \quad or \quad H_\Phi = \frac{I}{2\pi r}$$

The Biot-Savart law

- In Paris of 1840, Jean-Baptiste Biot and his assistant Felix Savart derived a law, which allows to calculate the *H*-field strength and direction from any conductor geometry and current, for any point in space. In integral form, it is…



$$\vec{H} = \frac{I_1}{4\pi} \cdot \oint_S \frac{d\,\vec{s} \times \vec{r}}{|r|^3}$$

- For practical applications, the integral is problematic, as closed analytical solutions only exist for special cases (like for a circular conductor).
- The original Biot-Savart law does not take wave propagation into account, so there is an error with increasing distance to the conductor.

Analytical formulas for specific geometries

Circular planar loop

- For the *H*-field magnitude on the center axis of a "short cylindric coil" we find

$$H = \frac{I \cdot N \cdot R^2}{2\sqrt{\left(R^2 + x^2\right)^3}}$$

H.….Field strength at point in space

I..….Antenna current (rms)

N..…number of turns

R.....(average) radius of coil

x......distance on center axis

Rectangular planar conductor loop

- For the *H*-field magnitude on the center axis of a rectangular, planar loop with side lengths *a* and *b*, we find

$$H = \frac{I \cdot N \cdot ab}{4\pi \cdot \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 + x^2}} \cdot \left[\frac{1}{\left(\frac{a}{2}\right)^2 + x^2} + \frac{1}{\left(\frac{b}{2}\right)^2 + x^2}\right]$$

H.….Field strength at measurement point in space

I..….Antenna current (rms)

N..…number of turns

R.....(average) radius of coil

x......distance on center axis

- Frame condition: planar (or short) coil (length << radius) and near-field (x << l/2π)

Biot-Savart law for circular loops extended by retardation (valid estimation for near-field & far-field)

– The radius vector from source (S, loop center) to any point in space (R) is…

$$r_{SR}(\Phi, x_R, y_R, z_R) = \sqrt{(x_S + a\cdot\cos(\Phi) - x_R)^2 + (y_S + a\cdot\sin(\Phi) - y_R)^2 + (z_S - z_R)^2}$$

– Magnitudes of the cartesian *H*-field components are…

$$\left|H_z(x_R, y_R, z_R)\right| = \frac{I_A\cdot a}{4\cdot\pi}\cdot\left|\int_0^{2\pi}\left\{\frac{e^{-i\cdot\beta\cdot r_{SR}}}{r_{SR}{}^2}\cdot\left(i\cdot\beta + \frac{1}{r_{SR}}\right)\cdot\left[a + (x_S - x_R)\cdot\cos(\Phi) + (y_S - y_R)\cdot\sin(\Phi)\right]\right\} d\Phi\right|$$

$$\left|H_x(x_R, y_R, z_R)\right| = \frac{I_A\cdot a\cdot(z_S - z_R)}{4\cdot\pi}\cdot\left|\int_0^{2\pi}\left\{\cos(\Phi)\cdot\frac{e^{-i\cdot\beta\cdot r_{SR}}}{r_{SR}{}^2}\cdot\left(i\cdot\beta + \frac{1}{r_{SR}}\right)\right\} d\Phi\right|$$

$$\left|H_y(x_R, y_R, z_R)\right| = \frac{I_A\cdot a\cdot(z_S - z_R)}{4\cdot\pi}\cdot\left|\int_0^{2\pi}\left\{\sin(\Phi)\cdot\frac{e^{-i\cdot\beta\cdot r_{SR}}}{r_{SR}{}^2}\cdot\left(i\cdot\beta + \frac{1}{r_{SR}}\right)\right\} d\Phi\right|$$

– H_Z is especially useful for a coaxial reader – card coupling scenario

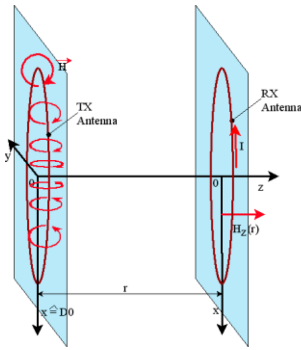
H in Near Field and Far Field, coaxial and coplanar

The *H*-field decreases with distance to the emitting loop antenna in...

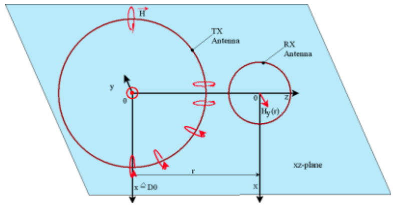
➔ **near-field:**

coaxial & coplanar orientation: H~1/d³

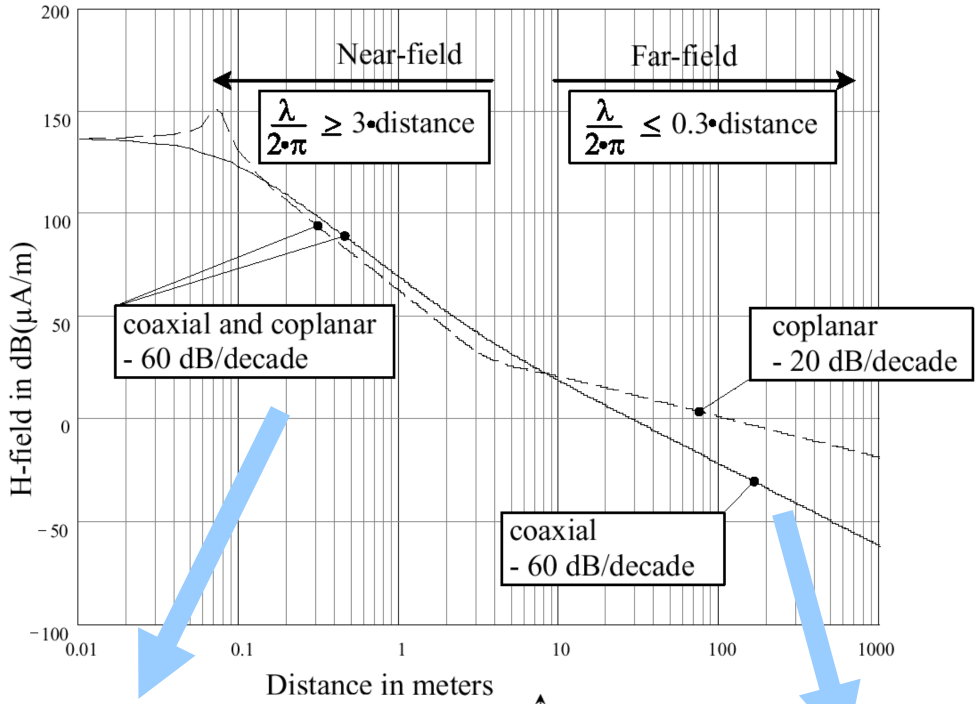
➔ **far-field:**

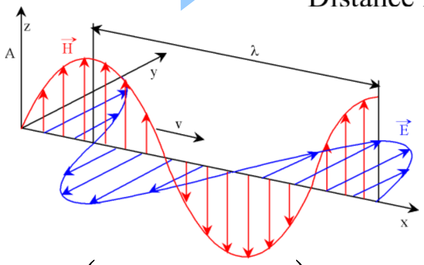


coaxial: H ~ 1/d³



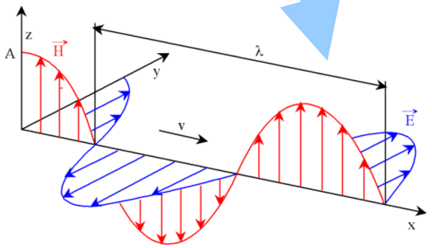
coplanar: H ~ 1/d¹





$$\angle\left(E(t),H(t)\right)\neq 0°$$

reactive power region



$$\angle\left(E(t),H(t)\right)= 0°$$

effective power region

Two considerations using the Biot-Savart law

- The Biot-Savart law gives accurate results for the Near Field and takes into account the antenna geometry. So we will use it to make two considerations, which are essential for size and shape of the loop antenna:
  - **Homogeneity** of the emitted *H*-field
  - **Optimum** (circular) antenna **radius**
- The *H*-field can be described by a vector, which defines magnitude (strength) and direction of the field, for any point in space. So in cartesian coordinates there are 3 components, x, y, z. Of special interest is the case, where z is the loop antenna axis, and x = y = 0.
- Moreover, close to the conductor loop contributions of both conductors are in opposite direction and same strength, so they cancel out. Considering H_Z across a plane over the loop antenna, one can find a *H*-field magnitude decrease over the antenna center in short distance, and an increase in higher distance. In between there is a specific distance to the antenna, where the *H*-field is equal (homogenous). This distance is related to the antenna radius.

Optimum Loop Antenna radius

- If we vary the loop antenna radius of the emitting antenna, for a fix distance to the receive antenna in a coaxial antenna arrangement, we can find a maximum of *H*-field. Of course, this maximum applies only for this fix distance.
- So, there is an optimum loop antenna radius, related to the intended distance regarding *H*-field emission. We can calculate the derivative of the equation for *H*-field of circular antennas, to find this optimum radius:

$$H'(R) = \frac{d}{dR}\left(H(R)\right) = \frac{2INR}{\sqrt{\left(R^2 + x^2\right)^3}} - \frac{3INR^3}{\left(R^2 + x^2\right)\cdot\sqrt{\left(R^2 + x^2\right)^3}}$$

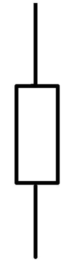
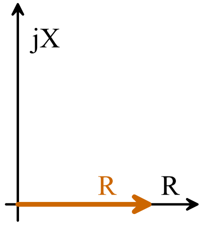
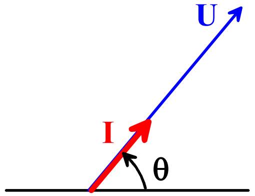
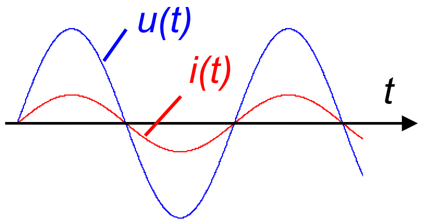
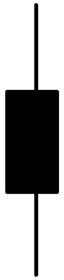
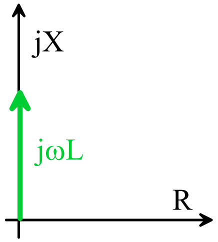
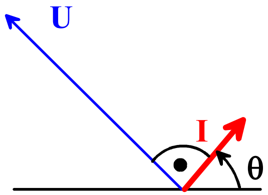
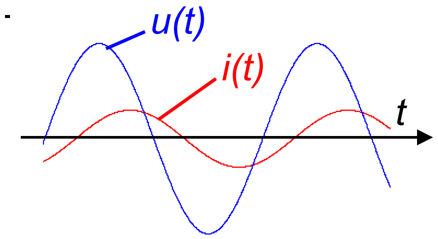
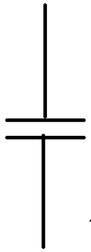
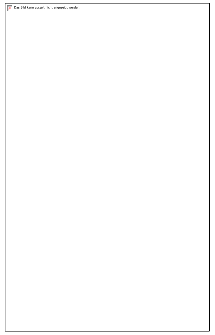
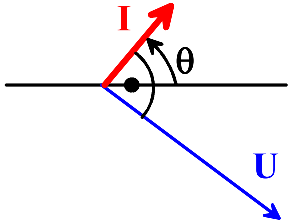
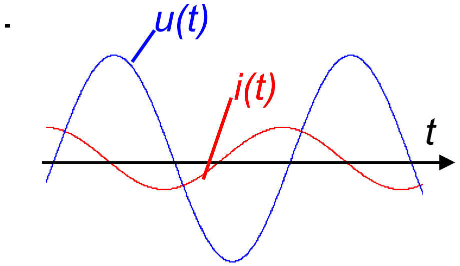
- Zeros of this function are at $\pm x \cdot \sqrt{2}$
- So, the optimum radius regarding RF power requirements for *H*-field emission is roughly 1,4 x the distance to the loop antenna center.
- As a rule of thumb, the maximum distance should be roughly equal to the loop antenna diameter.

Electrical Elements

Network elements

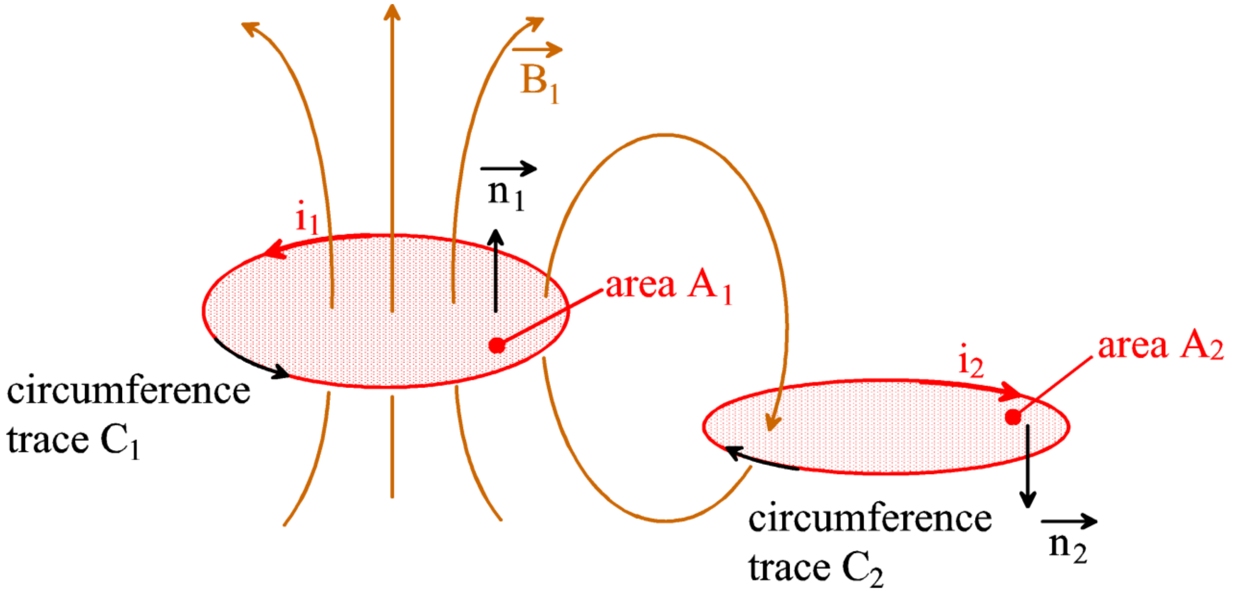
Overview of electrical elements

$Z = R + jX \qquad Y = G + jB$

symbol		impedance	phasor	signals	signal traces
	$Z = R$ $Z = R$			$i(t) = I_0 \cos(\omega t + \varphi_0)$ $u(t) = U_0 \cos(\omega t + \varphi_0)$	
	$Z = sL$ $Z = j\omega L$			$i(t) = I_0 \cos(\omega t + \varphi_0)$ $u(t) = U_0 \cos(\omega t + \varphi_0 + 90°)$	
	$Z = \frac{1}{sC}$ $Z = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$			$i(t) = I_0 \cos(\omega t + \varphi_0)$ $u(t) = U_0 \cos(\omega t + \varphi_0 - 90°)$	

What is the Element Inductance?

- Let us consider two current-carrying conductor loops, and time-variant current:



C_1…..circumference of 1st area

A_1…..area enclosed by the 1st conductor

i_1……current in the 1st conductor

$\vec{n}_1$……normal vector to the area A_1

$\vec{B}$……magnetic flux density vector

  - The current i_1 in the first loop generates a magnetic flux Φ of the flux density B_1.
  - A part of the primary flux penetrates the second conductor loop.
  - This generates a 2nd current, which compensates the part of the primary flux.

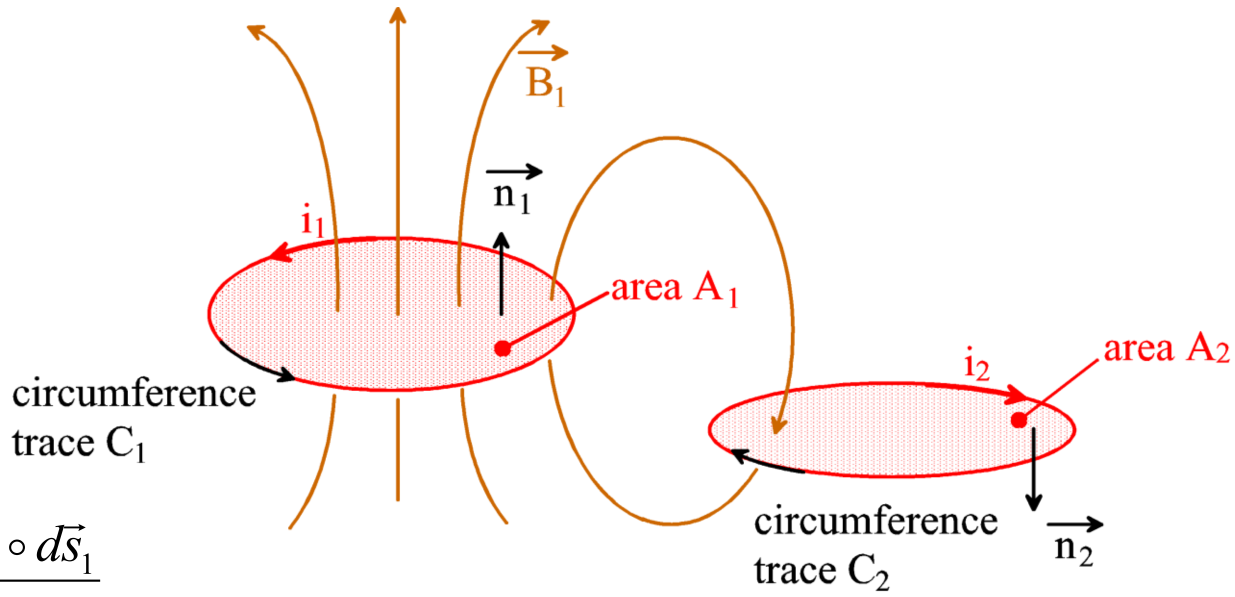
$$\Phi_2 = \iint_{A_2} \vec{B} \circ \vec{n}_2 dA_2 = \oint_{C_2} \vec{A}(P_2) \circ d\vec{s}_2 \quad for \quad \vec{B} = rot\vec{A} \quad \vec{A} \text{.....magnetic vector potential}$$

What is the Element Inductance?

- All contributions to the flux across an area are directly proportional to the currents in the individual conductor loops, e.g.

$$\Phi_2 = I_1 L_{12} + I_2 L_{22}$$

  - L with same indices means self-inductance.
  - L with different indices means mutual inductance.



- Resolving the proportionality value, we find
  - For the mutual inductance…

$$L_{12} = L_{21} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{s}_2 \circ d\vec{s}_1}{R_{12}}$$

  - For the self-inductance…

$$L_{22} = \frac{\mu_0}{4\pi I_2} \oint_{C_2} \iiint_{V_2} \frac{\vec{S}_2(P_2')}{R_{2'2}} dV_2 \circ d\vec{s}_2$$

Examples:

- Ferrite is a special material which has an increased relative permeability μ_R compared to free space (where μ_R = 1), the flux density *B* is increased.
- Consequently, Inductance *L* is increased for a conductor loop near ferrite.

$$B = \mu_R \cdot \mu_0 \cdot H \quad where \quad \mu_0 = 4\pi 10^{-7} \frac{V\,m}{A\,s}$$

- As metal allows ring currents (eddy currents) equal to a closed conductor loop, Inductance *L* is decreased for a conductor loop near metal.

What is the Element Inductance?

- We can understand inductance also as "inertia of current", it is the time-variant "resistance" which the conductor offers to a time-variant current.
- Inductance results from the relation of time-variant magnetic flux and current to

$$L = \frac{d\Phi}{dI}$$ …or for the coil flux (*N* turns) $$\psi = N\Phi$$

- All the magnetic flux Φ generated by the current *i* is directly proportional to the actual value of *i*. The proportional value is inductance *L:*

$$u_i(t) = -\frac{d\psi(t)}{dt} = -N\frac{d\Phi}{di}\cdot\frac{di}{dt} = -L\frac{di}{dt} \quad \rightarrow \quad L = -\frac{d\psi}{di} = -N\frac{d\Phi}{di}$$

- For harmonic sine-wave signals (considering no offset or transient condition) we can express the derivation by amplitudes, for complex network calculations:

$$U = I \cdot jX_L \quad for\ X_L = \omega L$$ …where *j* expresses the 90 ° phase-shift

Example: Mutual inductance of circular coils

- For the simple geometry of circular coils, an exact calculation is possible:

$$M_C = \frac{\mu_0 \cdot r_1 \cdot r_2}{2} \int_0^{2\pi} \frac{\sin(\alpha)}{\sqrt{x^2 + r_1^{\,2} + r_2^{\,2} - 2\, r_1\, r_2 \cos(\alpha)}}\, d\alpha$$

M_C…..mutual inductance in Henry (H)

μ_0…..permeability constant

r_1..….radius of the 1st coil in meters

r_2 ….radius of the 2nd coil in meters

x……distance of the coil centers in meters

α ..….tilt angle of the coil axes

- Moreover, for coaxial coil orientation (same axis), this simplifies to

$$M_{CA} = \mu_0 \frac{N_1\, r_1^{\,2}\, N_2\, r_2^{\,2}\, \pi}{2\sqrt{\left(r_1^{\,2} + x^2\right)^3}}$$

Coupling factor

- In network calculation, the coupling factor *k* represents the connection of a coil arrangement.
- It is a pure geometry factor, as the other parameters cancel out.
- It results from the relation of mutual inductance *M* between two coils, and the inductance *L* of the two coils:

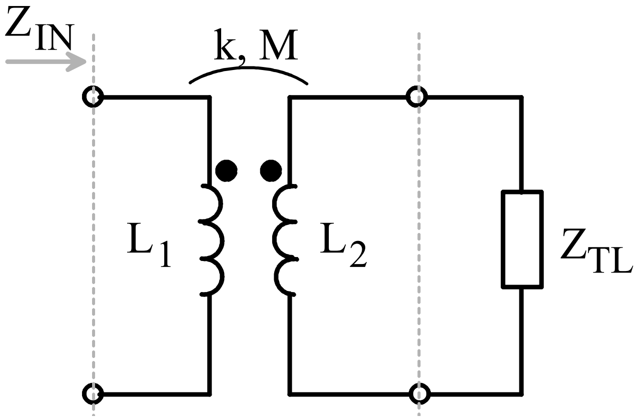
$$k = \frac{M}{\sqrt{L_1 \cdot L_2}} \qquad for\ M_{12} = M_{21} = M$$

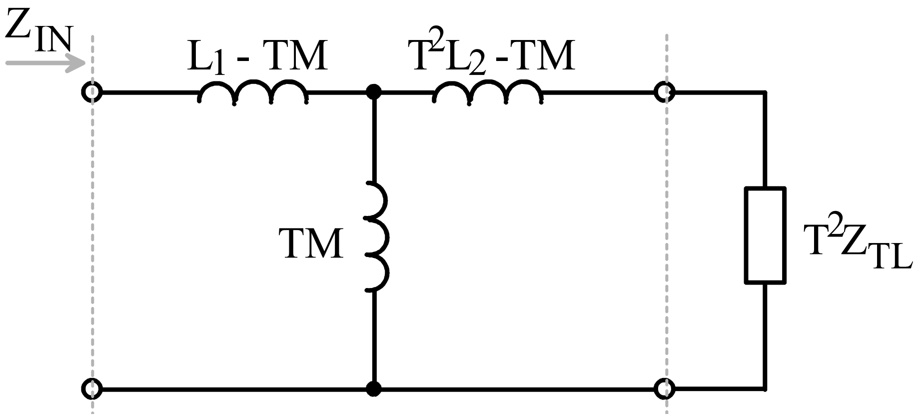
- For example, resolving the equation for *circular coils in coaxial orientation*, we find…

$$k = \frac{r_1^2 \cdot r_2^2}{\sqrt{r_1 r_2} \cdot \sqrt{\left(r_1^2 + x^2\right)^3}}$$

Representing Coupling & Transformation

- Coupled inductances can be represented by an equivalent T – structure.





$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$T = \sqrt{\frac{L_1}{L_2}}$$

k.....coupling

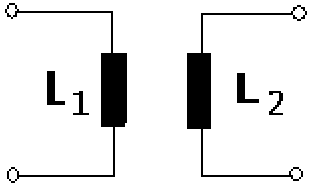
M....mutual inductance

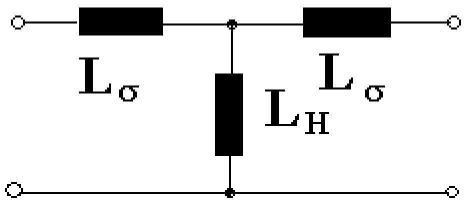
T.....Transformation

Note: *ideal coupling k = 1 assumed*

Measurement of mutual inductance and coupling

- An exact calculation from coil geometry is not always possible, sometimes a measurement is easier to do (also to check a calculation). One option uses an LCR meter and gives an approximation for mutual inductance:





L_σ......strey inductance

L_H......main inductance

1. Open measurement:
   - The 1st coil is connected to the LCR-meter, the 2nd coil remains open.

$$L_{MEAS} = L_\sigma + L_H \approx L_H$$

2. Short measurement:
   - The 1st coil is connected to the instrument, 2nd coil is shorted. Measure *L* @ 13,56 MHz

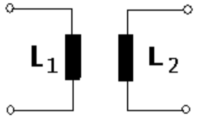
$$L_{MEAS} = L_\sigma + \left(L_\sigma // L_H\right) \approx 2L_\sigma$$

The strey factor is $\sigma = \frac{L_{SHORT}}{L_{OPEN}}$ *and the coupling factor is* $k = \sqrt{1-\sigma^2} = \sqrt{1 - L_{SHORT} / L_{OPEN}}$

Mutual inductance M is given by 

Measurement of coupling (alternative)

- Another practical method to measure the coupling factor *k* results from a simple consideration for a non-ideal transformer:



$$\frac{U_2}{U_1} \cong k \cdot \frac{N_2}{N_1} \quad where \quad L \sim N^2$$

- If we apply AC voltage on a primary coil and measure induced voltage on a secondary coil, coupling can be derived from

$$k \cong K_F \cdot \frac{U_2}{U_1} \cdot \sqrt{\frac{L_1}{L_2}}$$

k..........coupling factor

K_F.......correction factor (< 1)

U_1.......primary coil voltage

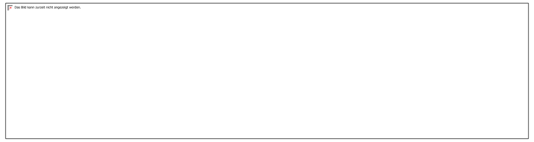
U_2........secondary coil voltage

L_1........Inductance, primary coil

L_2........Inductance, secondary coil

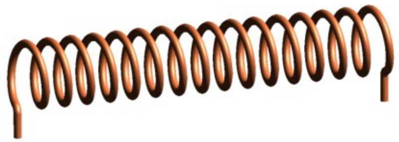
  – Main frame condition is, that the current in the 2nd coil shall be negligible, to avoid a voltage drop on the coil resistance and to avoid any loading on the 1st coil. The best is, to do such measurement with an active probe, or a voltage follower with low input cap. The effect of a parasitic cap can be taken into account by a correction factor.

$$C_{GES} = C_2 + C_{PROBE}$$



- The same concept can, of course, also be used to simulate mutual inductance in circuit simulators (e.g. Spice).
  It is, however, an approximation only.

What is the element Inductance?



- Inductance (L) is a property of conductors and coils, relating time-variant voltages to currents.

$$L = \frac{N\Phi}{I}$$ …in *Henry (H)*

…for the long coil $L = N^2 \frac{\mu_R \mu_0 A}{l}$

…for the straight conductor $L = \frac{\mu_R \mu_0 l}{8\pi}$

…where μ_0 is the permeability constant $\mu_0 = 4\pi \cdot 10^{-7}\, {}^{Vs}\!/_{Am}$

- Inductance also is an energy (W) storage and thus can be defined

$$W_M = \frac{L\hat{i}_I^{\,2}}{2} = L I_{RMS}^{\;2}$$

- Complex network calculation (without pre-charge)

$$u(t) = L\frac{di(t)}{dt} \qquad U = I \cdot sL$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} u(t)dt \qquad I = \frac{U}{sL}$$

What is the element Capacitance?

- Capacitance (C) is the ability of a body to store an electrical charge (*q*). Typically two conductive plates of area *A* in distance *d* store the charge +*q* and –*q*. Capacitance is then given by

$$C = \frac{q}{U}$$ …in *Farad (F)*

…for the plate capacitor $C = \varepsilon_R \varepsilon_0 \frac{A}{d}$

…where ε_0 is the electric field constant $\varepsilon_0 = 8{,}854 \cdot 10^{-12}\, F/m$

- Capacitance also is an energy (*W*) storage and thus can be defined

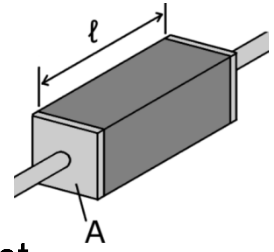
$$W_E = \frac{C\hat{u}_C^{\,2}}{2} = C U_{RMS}^{\,2}$$

- Complex network calculation (without pre-charge)

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt \qquad U = I \cdot \frac{1}{sC}$$

$$i(t) = C\frac{du(t)}{dt} \qquad I = U \cdot sC$$

What is the element Resistance?



- Resistance (R) of an electric conductor represents the loss of effective power, when the conductor carries current.

  Conductor materials have a specific conductance σ in S/m. Resistance is given by

  $$R = \frac{l}{\sigma \cdot A}$$ …in *Ohm* (Ω)

- Resistance also means loss power (P)

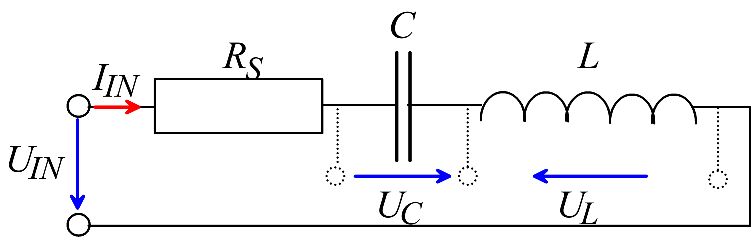
  $$P = U \cdot I = \frac{U^2}{R} = I^2 \cdot R = \mathrm{Re}\{W\} = \mathrm{Re}\{u(t) \cdot i(t)\}$$

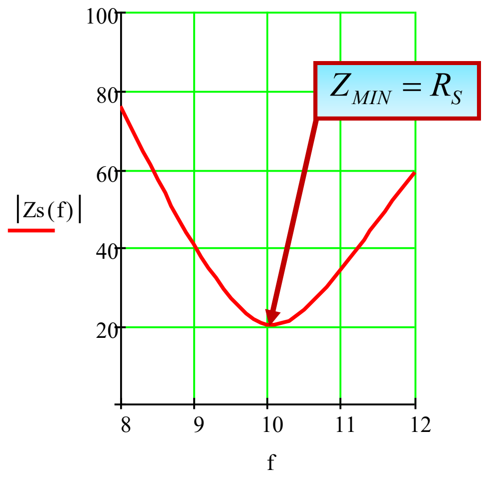
- Complex network calculation (there cannot be any pre-charge)

$$u(t) = R \cdot i(t) \qquad U = I \cdot R$$

$$i(t) = \frac{u(t)}{R} \qquad I = \frac{U}{R}$$

Resonance circuits – serial resonance





– Voltage resonance
– Absorption circuit ("Saugkreis")

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

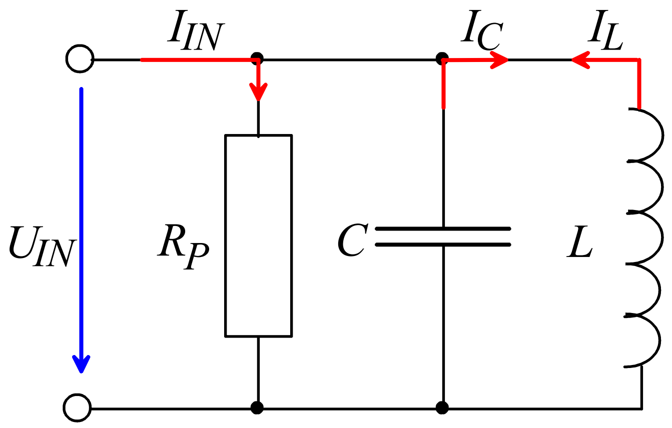
$$Q = \frac{\omega_{RES} L}{R_S} = \frac{1}{\omega_{RES} R C} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

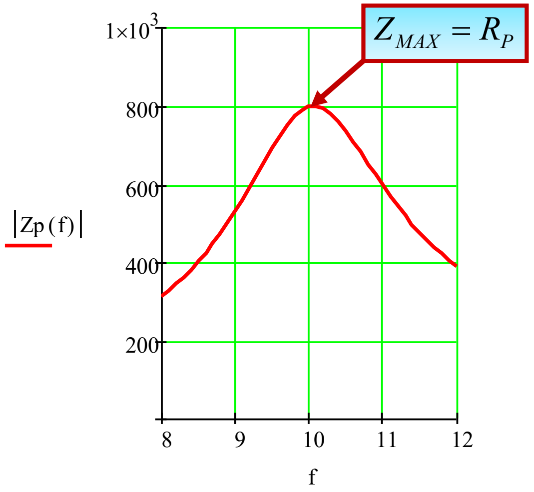
$$f_{RES} = \frac{1}{2\pi\sqrt{LC}}$$

$$U_L(@f_{RES}) = U_C(@f_{RES}) = QU_{IN}$$

$$I_{MAX}(@f_{RES}) = \frac{U_{IN}}{R_S}$$

Resonance circuits – parallel resonance





- Current resonance
- Trap circuit ("Sperrkreis")

$$|Z| = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

$$f_{RES} = \frac{1}{2\pi\sqrt{LC}}$$

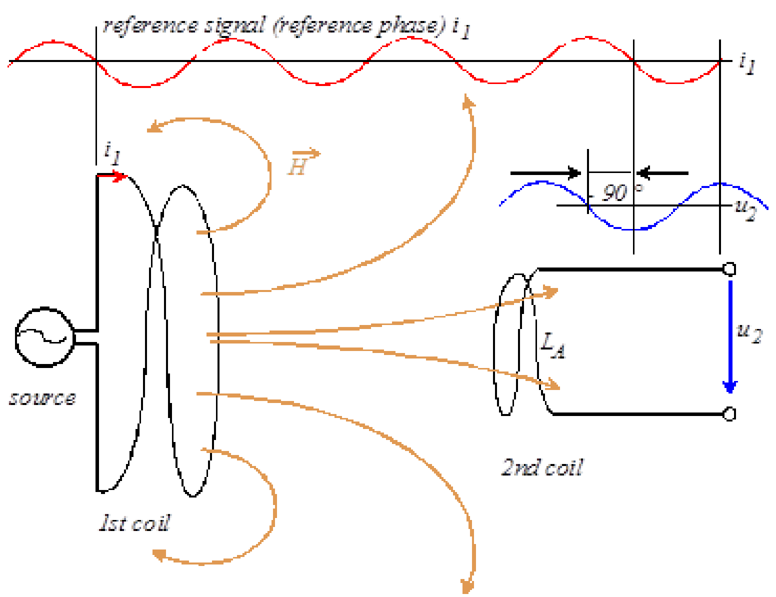
$$Q = \frac{R_P}{\omega_{RES} L} = \omega_{RES} R C = R\sqrt{\frac{C}{L}}$$

$$U_{MAX}(@ f_{RES}) = R_P \cdot I_{IN}$$

$$I_C(@ f_{RES}) = I_L(@ f_{RES}) = Q\, I_{IN}$$

Coupling case 1: Open Loop

- Induced voltage is related to primary *H*-field
- No current in 2nd loop, so no *H*-field emission



Maxwells Equations, integral form

(1) $$\oint_C \vec{H} \circ d\vec{s} = \iint_A \vec{J} \circ \vec{n}\, dA + \frac{d}{dt} \iint_A \vec{D} \circ \vec{n}\, dA$$

(2) $$\oint_C \vec{E} \circ d\vec{s} = -\frac{d}{dt} \iint_A \vec{B} \circ \vec{n}\, dA$$

(3) $$\oiint_A \vec{B} \circ \vec{n}\, dA = 0$$

(4) $$\oiint_A \vec{D} \circ \vec{n}\, dA = \iiint_V \rho\, dV$$

Michael Faraday, Law of Induction (1831)

$$\oint_C \vec{E} \circ d\vec{s} = u_i = -\frac{d\Phi_m}{dt}$$

Harmonic sine-wave flux (quasi-stationary)

$$U_i = -\frac{d\varphi}{dt} = -\frac{d(B\,A)}{dt} = -\frac{d(\mu\, H\, A)}{dt} = -\frac{dH}{dt} \cdot \mu_0 A$$

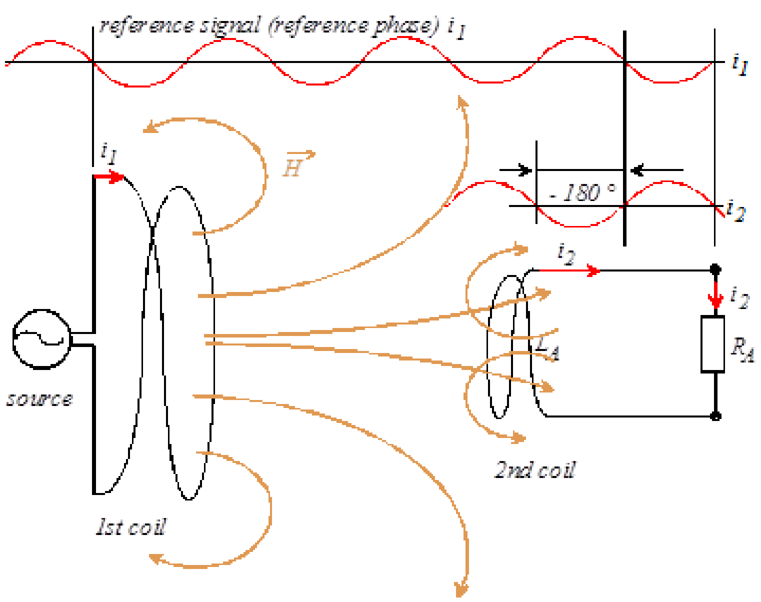
$$H(t) = H \cos(\omega t) \;\rightarrow\; -\frac{dH(t)}{dt} = \omega\, H \sin(\omega t)$$

$$|U_i| = 2\pi\, f \cdot \mu_0\, H\, A$$

Ref.: 13th International Conference on Telecommunications (ConTEL), Graz, Austria

Coupling case 2: Closed Loop

- Rule of Lenz:
- Current in 2nd loop generates *H*-field…
- …that cancels out primary *H*-field (-180 °) at the position of the 2nd coil



Maxwells equations, differential form

$$(1)\quad rot\vec{H} = \vec{J} + \frac{\partial}{\partial t}\vec{D}$$

$$(2)\quad rot\vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

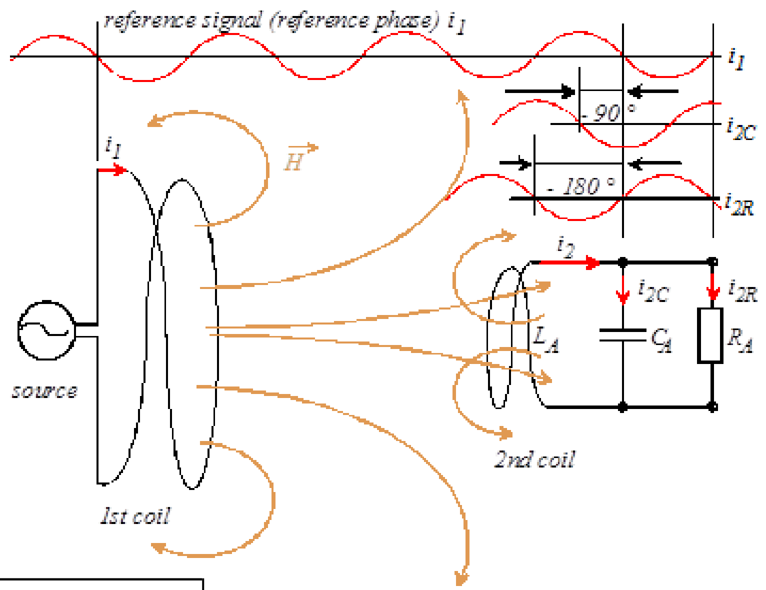
$$(3)\quad div\vec{B} = 0$$

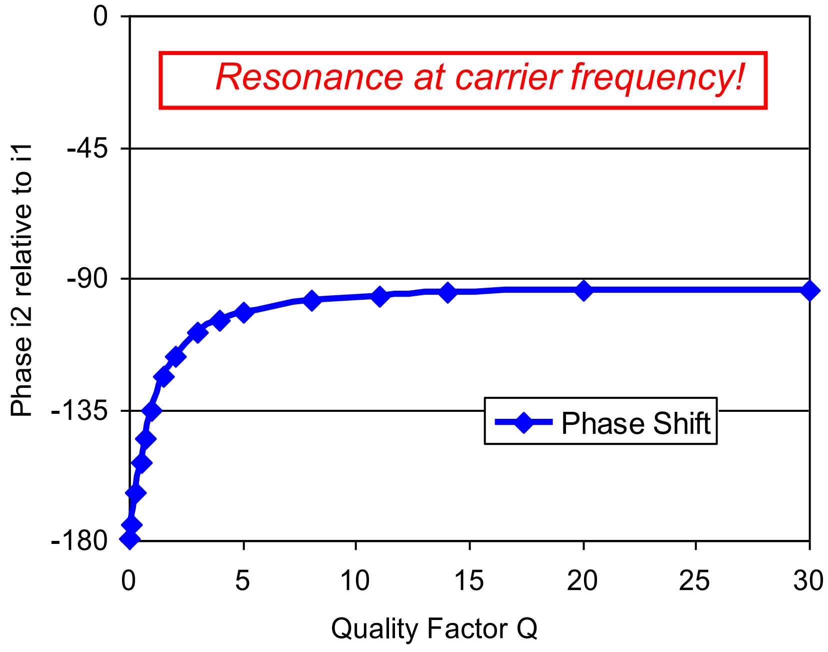
$$(4)\quad div\vec{D} = \rho$$

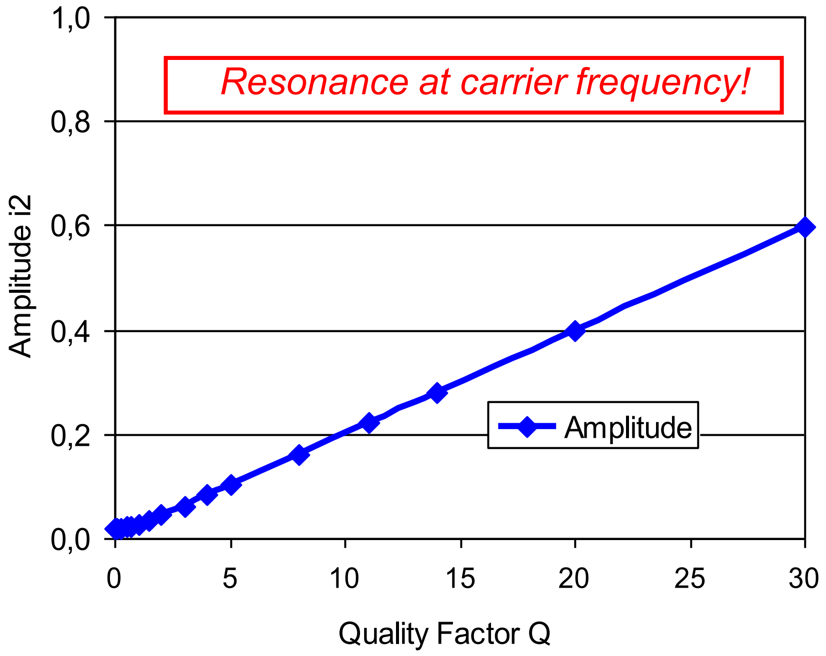
- A closed loop of ideal conductor shorts induced voltage to zero.
- This current in the 2nd loop is shifted by – 90 ° to an induced voltage (which is – 90 ° shifted to primary current), so in total the 2nd current is – 180 ° shifted versus the primary current.
- A resistance in series to the inductance allows some (induced) voltage drop – the remaining voltage is shorted and compensates (partly) the primary *H*-field at the second position.

Coupling case 3: Resonant loop antennas

- Current in 2nd loop is effective and reactive…
- Phase-shift is different from 180 °…
- Field does not cancel out,
- Amplitude of 2nd *H*-field can even exceed the primary field





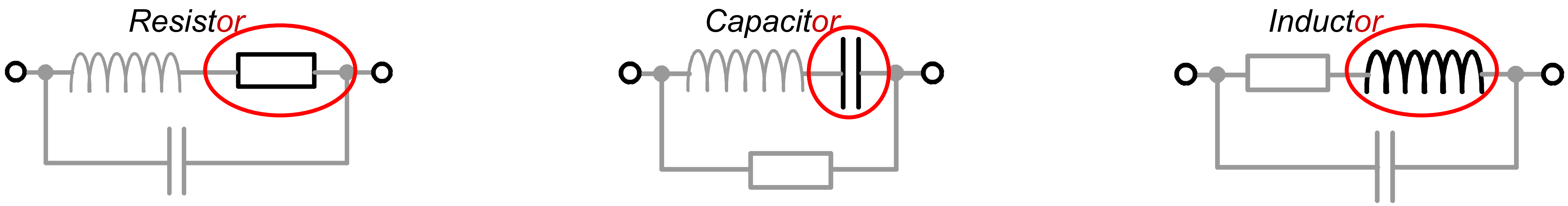


Electrical Components

Practical realization, how to measure the main property, and some dependencies

Electrical Components

- Components are the practical implementation of network elements. Main properties (elements) are associated with parasitic properties:



- Surface mounted devices (SMD) have the least (minimum) parasitics
- Electrical dependencies
  – Frequency dependency (e.g. dispersion)
  – Power dependency (e.g. saturation), …
- Ambient (physical) dependencies
  – Temperature, humidity (e.g. aging), pressure, …

☞ Practical hint: Try to characterize under operating conditions!

Resistor

- SMD resistors offer an excellent representation of the element resistance.
- Values are available in logarithmic distance for a decade.
- E series resistor values specified in ISO 60063:
  - E3, E6, **E12**, E24, E48, E96 and E192.
- Individual values *k* are calculated by $k \cong \sqrt[n]{10^m}$

  where *n* specifies the number of elements per decade *m*.
  - e.g. E12: $\sqrt[12]{10^0} = 1, \quad \sqrt[12]{10^1} \cong 1.2, \quad \sqrt[12]{10^2} \cong 1.5, ..., \sqrt[12]{10^{10}} \cong 8.2$
- Package sizes is specified in inches
  - e.g. 1206, 0805, 0603, 0402, 0201

means 2,0 mm long and 1,25 mm wide

Package	Length	Width
	mm	mm
1206	3,2	1,6
.0805	2	1,25
.0603	1,6	0,8
.0402	1	0,7
.0201	0,5	0,3

Capacitor

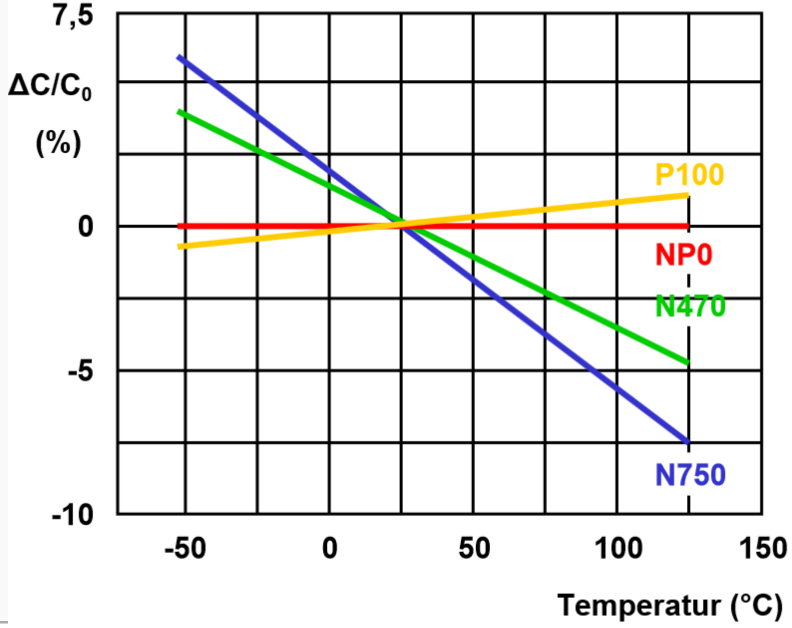
- Capacitors can be a similar good representation of the element capacitance, if the dielectric material is chosen right. COG or NP0 for SMD are good HF Caps.
- The Electronics Industries Alliance (EIA) has standardized 3 capacitor classes:
  - Class 1: HF capacitors (typically ceramic) with high parameter stability
  - Class 2: High volume efficiency capacitors (for buffers,…)
  - Class 3: Volume efficiency ceramic caps (typ. – 22…+ 56 % cap over 10...55 °C)
  - Class 4: Semiconductor caps
- Class 1 ceramic capacitors are classified for temperature dependency in a frequency range
  - IEC/EN 60384-8/24 means 2-digit code, EIA RS-198 means 3 digit code
  - NP0 means zero gradient and +/-15 x 106 / K tolerance. EIA code is C0G, IEC/EN code is C0
- The EIA ceased operations in 2011, the Electronic Components Industry Association (ECIA) will continue EIA standards maintenance.

Capacitor

Codierung von Temperaturkoeffizienten α und deren Toleranzen für Klasse-1-Keramikkondensatoren nach IEC/EN 60384-8/21 und Auflistung des entsprechenden EIA RS-198 Codes

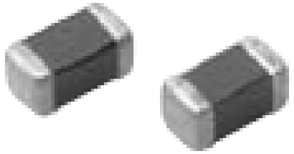
Bezeichnung	Temperatur-koeffizient α in 10^{-6} /K	IEC/EN-Code für α	α-Toleranz in 10^{-6} /K	IEC/EN-Code für α-Toleranz	Unter-klasse	IEC/EN-Code	EIA-Code
P100	100	A	±30	G *)	1B	AG	M7G
NP0	0	C	±30	G *)	1B	CG	C0G
N33	−33	H	±30	G *)	1B	HG	H2G
N75	−75	L	±30	G *)			
N150	−150	P	±60	H			
N220	−220	R	±60	H			
N330	−330	S	±60	H			
N470	-470	T	±60	H			
N750	−750	U	±120	J			
N1000	−1000	Q	±250	K			
N1500	−1500	V	±250	K			
+140 … −1000	–	SL	–	–			

*) Kennbuchstabe für α-Toleranz $\pm 15 \cdot 10^{-6}$ /K = F



Inductor

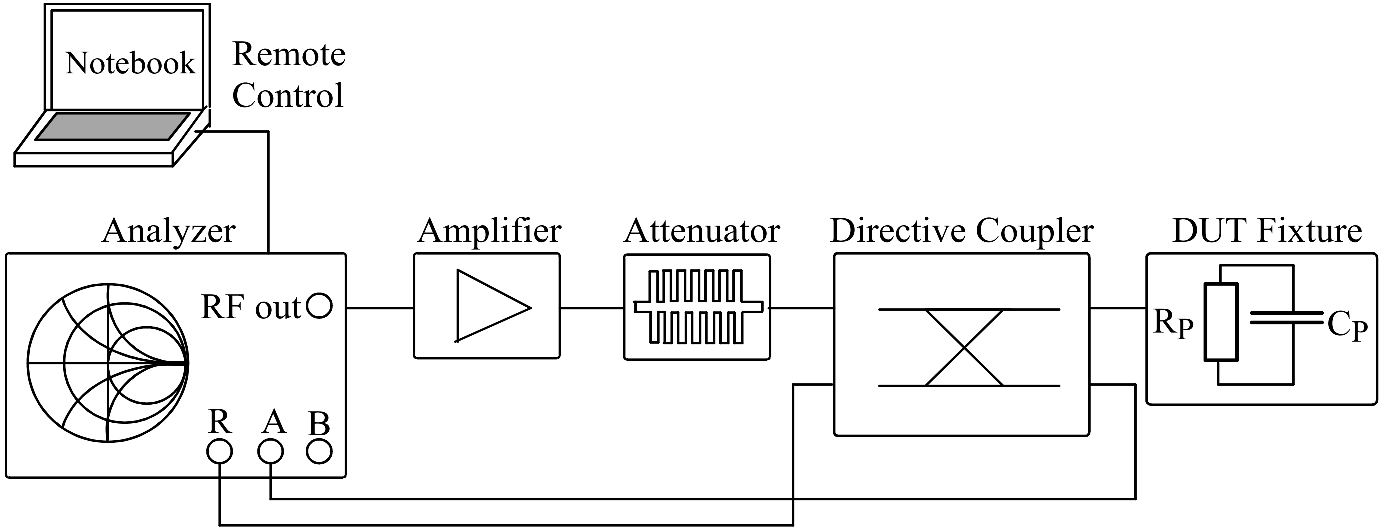
- Inductors are critical / problematic components.
- Wire-wound coil inductors preferred to chip inductors (more stable properties)



- Attention to the current under operating conditions (e.g. 100 mW ... 1 W RF power for matching networks)
- Losses due to parasitic DC resistance (e.g. 0,5 … 5 W for 1 µH in 0805 package) – reduces the Q-factor!
- Attention to frequency and power dependency of inductance
- Attention to thermal stress
- Take care of coupling in layout

How to characterize impedance at HF

- An extended setup for network analysis is used to characterize impedance over frequency and voltage - also in the operating point and up to destruction levels.



$$\underline{Z}_C = \frac{1+\underline{\Gamma}_C}{1-\underline{\Gamma}_C} Z_0 = \frac{1}{\underline{Y}_C}$$

$$R_P = \mathrm{Re}\{\underline{Z}_C\} = \mathrm{Re}\left\{\frac{1}{\underline{Y}_C}\right\} = \mathrm{Re}\left\{\frac{1}{G_P + jB_P}\right\} = \frac{1}{G_P}$$

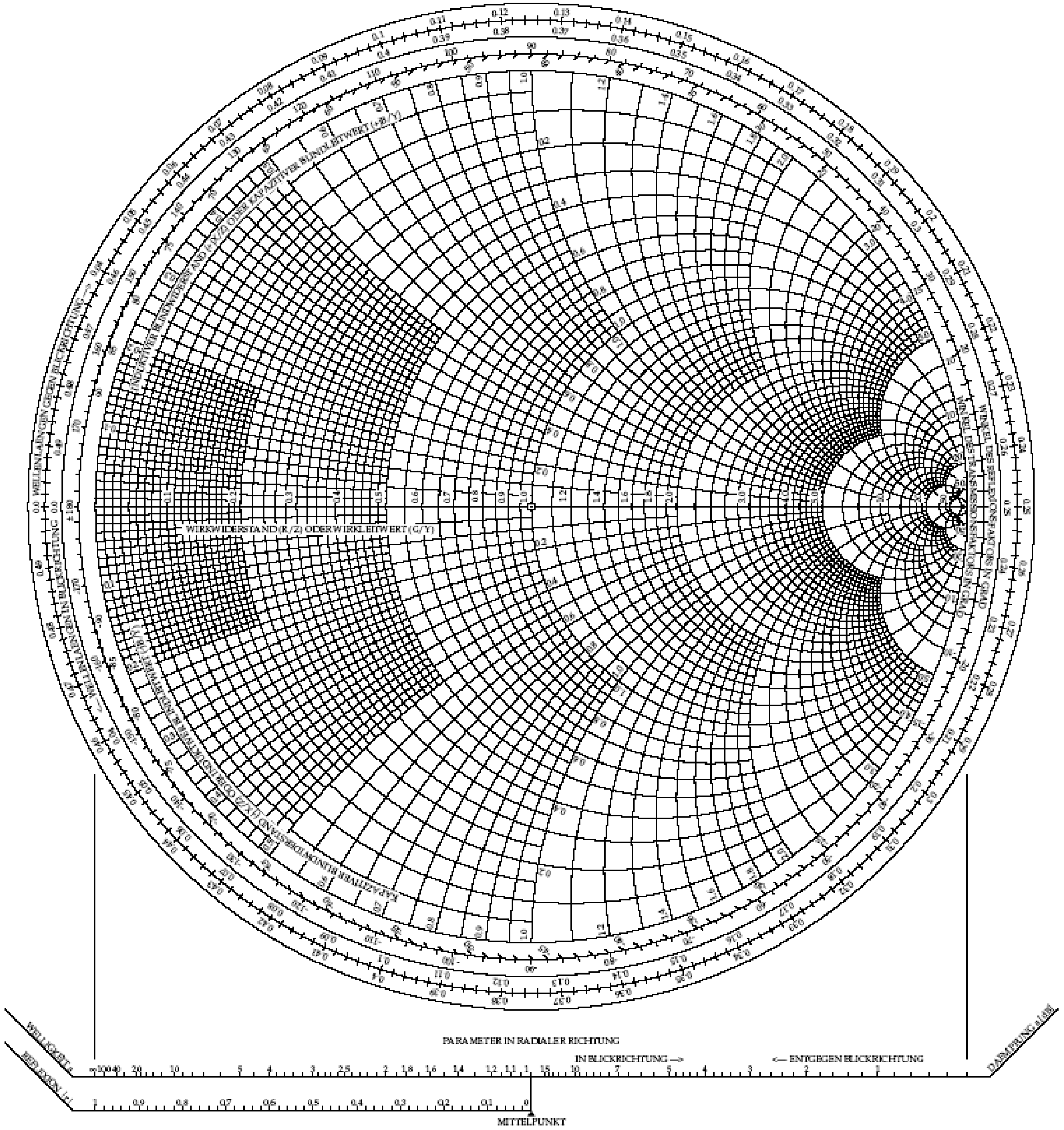
$$C_P = \mathrm{Im}\left\{\frac{1}{\underline{Y}_C}\right\} = -\mathrm{Im}\left\{\frac{1}{2\pi f_{MEAS} \underline{Z}_C}\right\} = \frac{B}{\omega}$$

- The voltage on the DUT is calculated with a voltage divider (50 Ohm source and measured load impedance), from a previously measured output voltage to 50 Ohms.

$$U_{DUT} = U_S \frac{R_P}{\sqrt{(R_P + R_S)^2 + (\omega R_P R_S C_P)^2}}$$

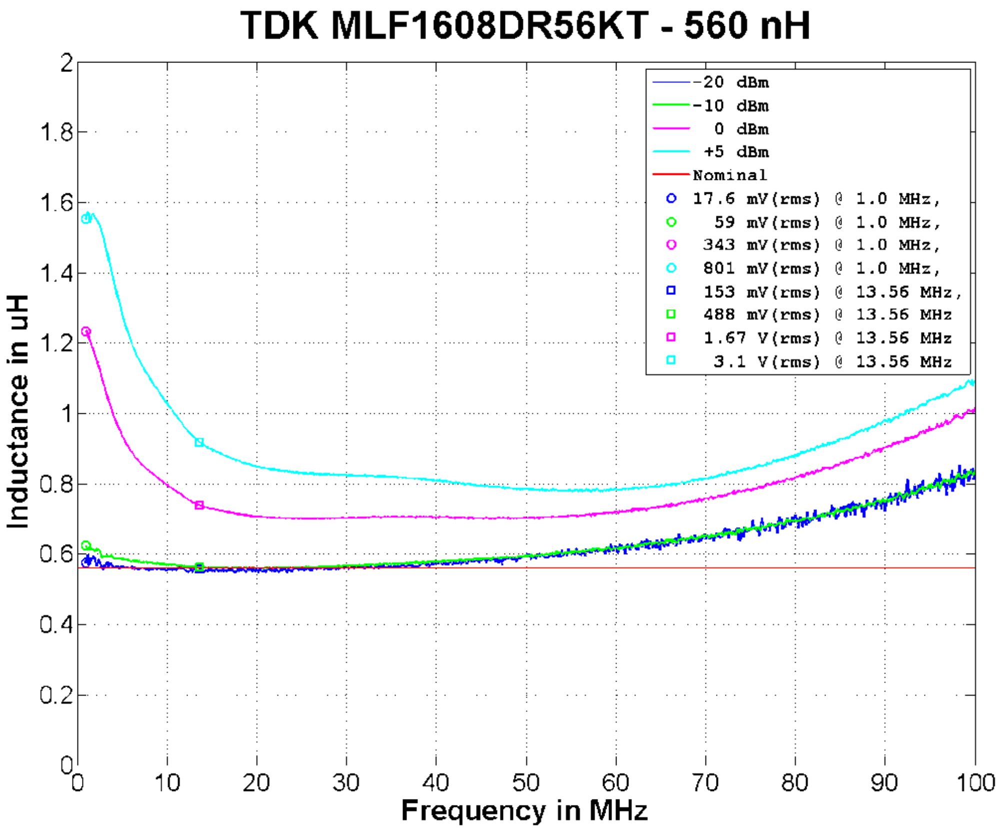
Smith Chart

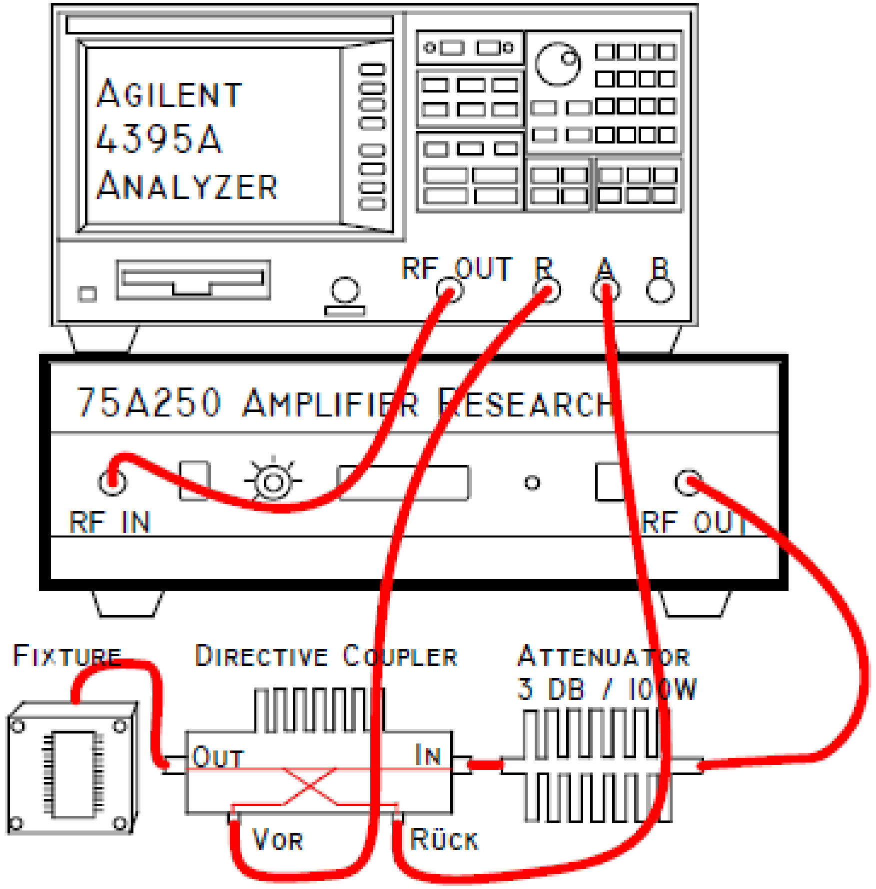
- The Smith chart was developed in 1939 by Phillip Smith.
- Background is impedance measurement at HF frequencies. This can be done by measuring the reflection by standing wave measurements. This is, how a network analyzer works. It makes sense, to use a display, which allows to read out impedance from measured reflection.
- The Smith chart is in the plane of the reflection coefficient Γ and represents a transformation of the complex impedance Z to Γ.
- It allows geometrical calculations, e.g. for impedance matching.



Example – Chip Inductor (560 nH)

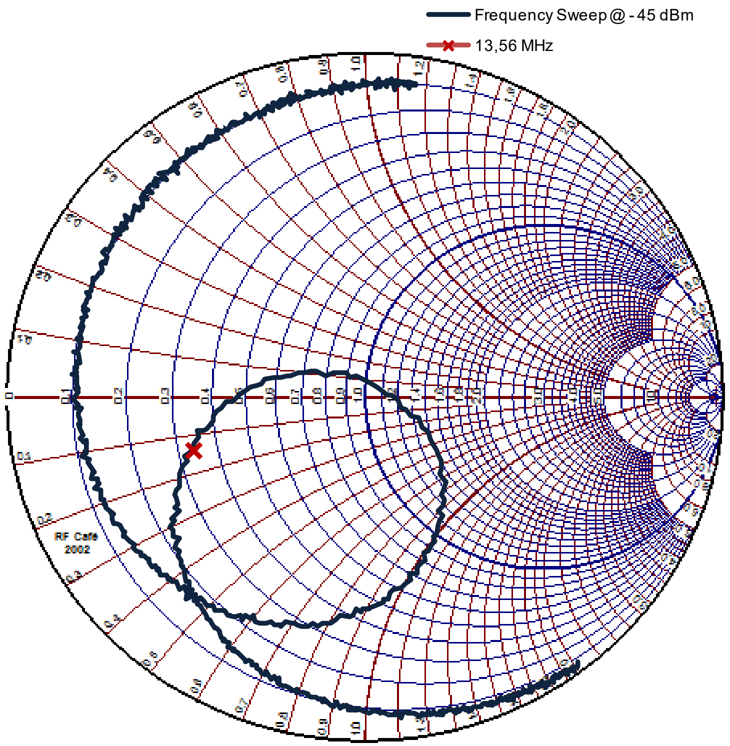
- In order to possibly extract an equivalent circuit of the inductor, a frequency sweep of the inductance was made at 4 different power levels.

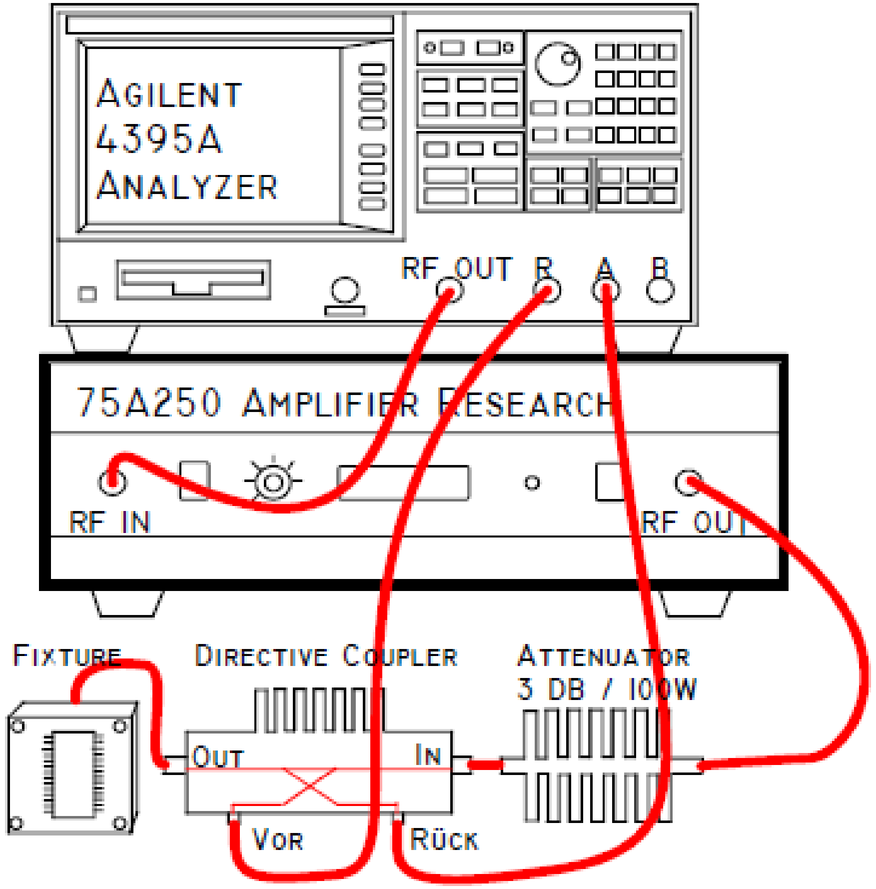




Measurement of the matching impedance

Smith Chart frequency sweep for power steps

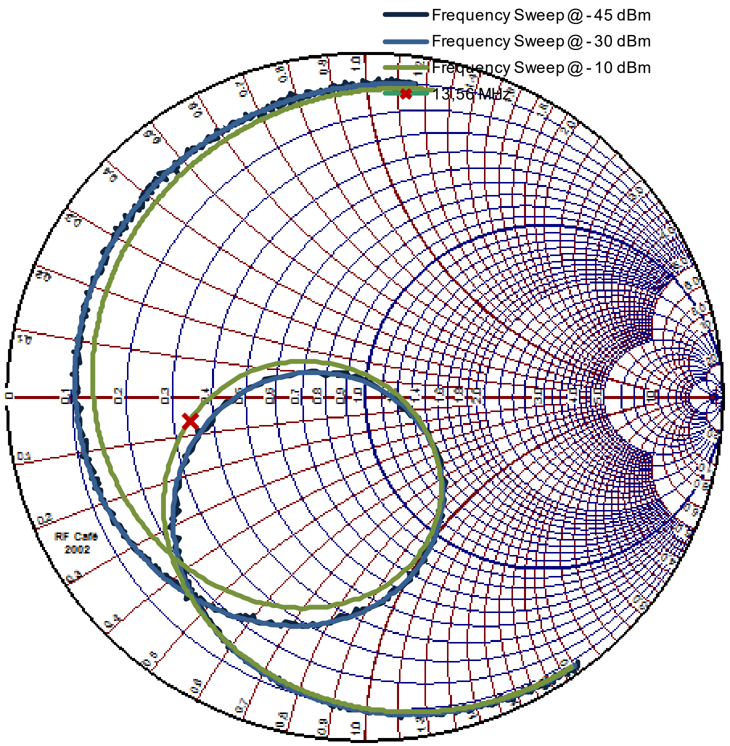


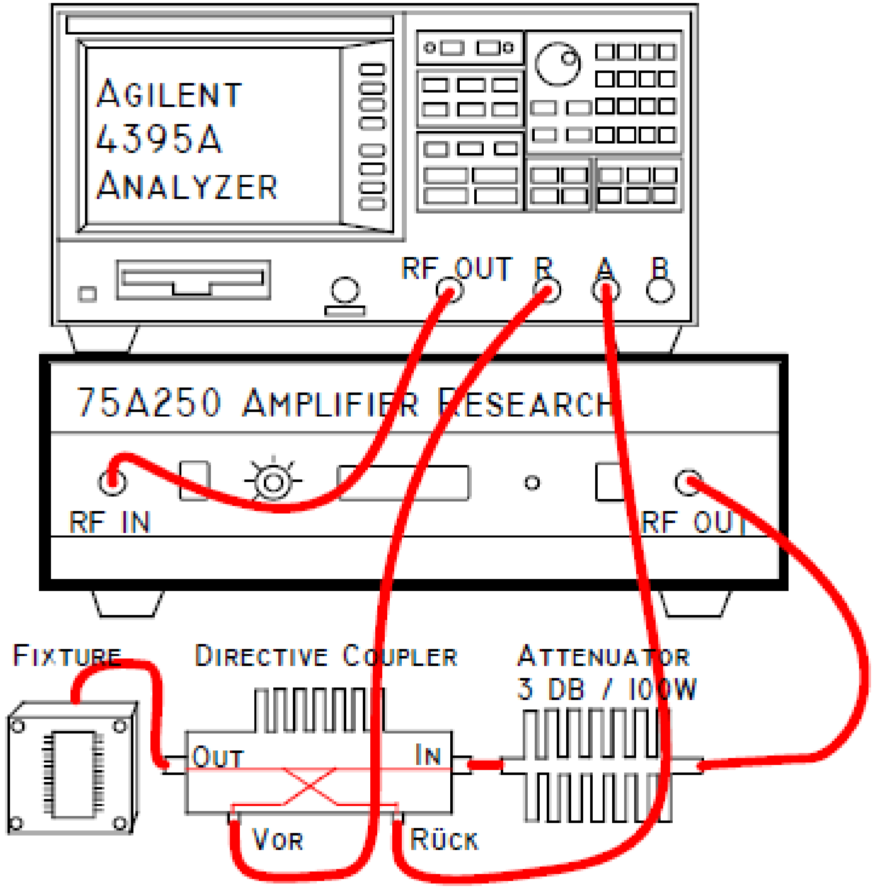


☞ Voltage @ 13,56 MHz in Matching NW is < 0,01V(RMS)

Measurement of the matching impedance

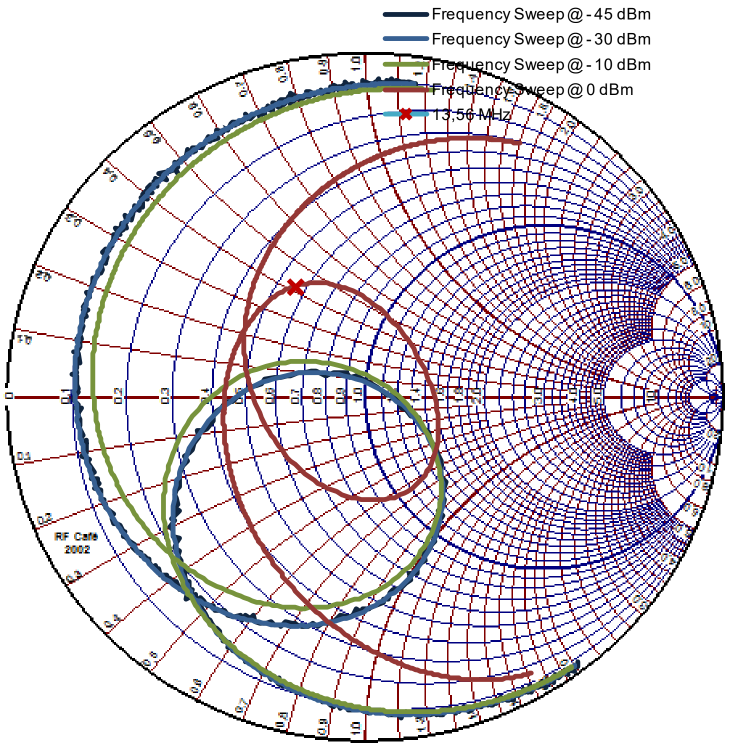
Smith Chart frequency sweep for power steps

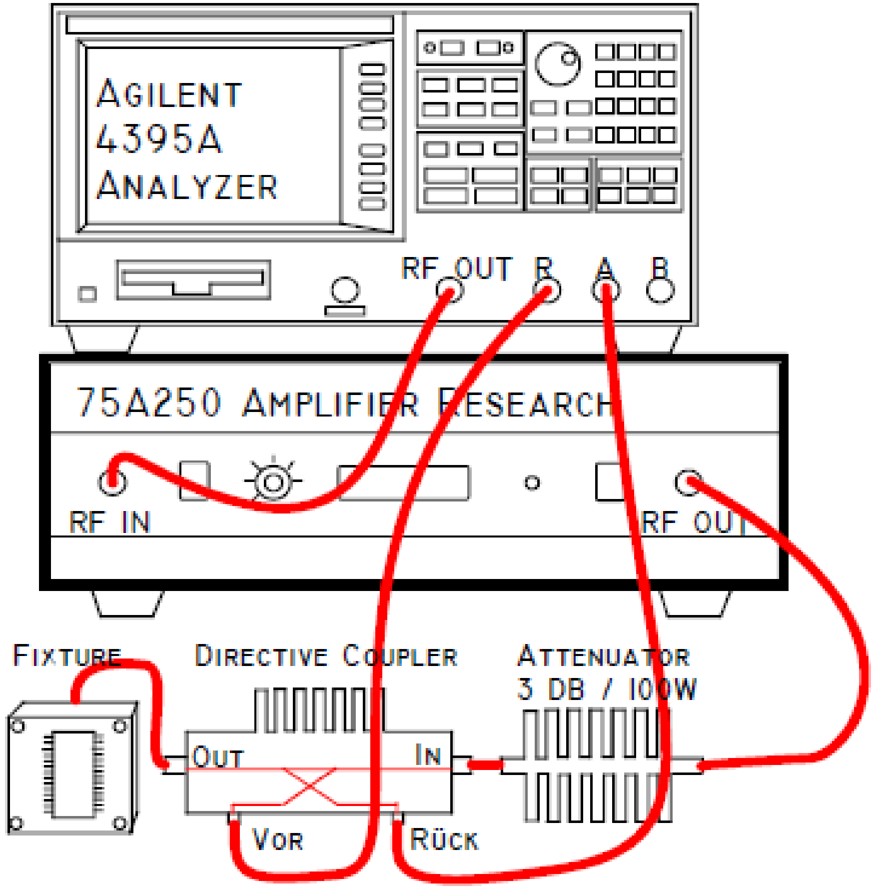




☞ Voltage @ 13,56 MHz in Matching NW is 0,133 $V_{(RMS)}$ = 0,934 $V_{(pp)}$

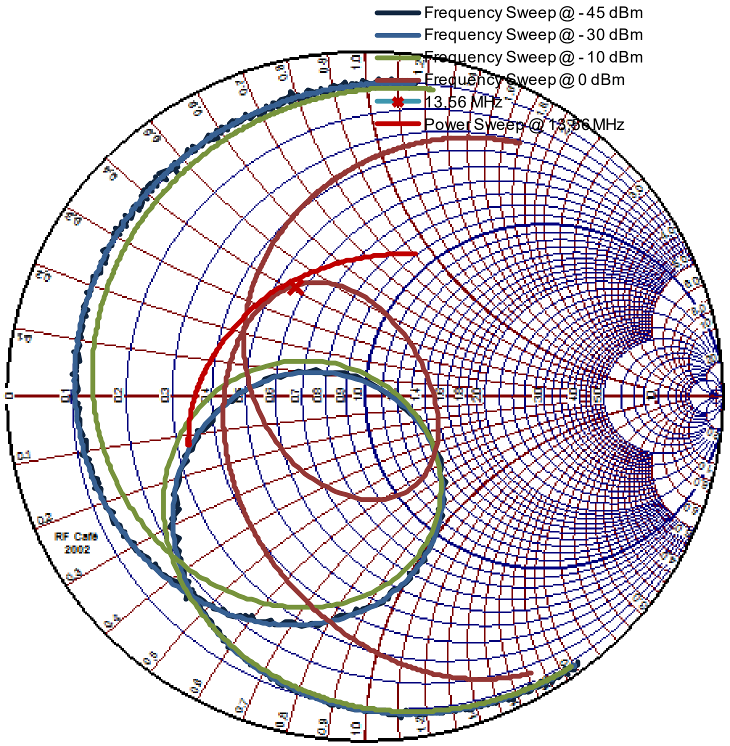
Measurement of the matching impedance
Smith Chart frequency sweep for power steps

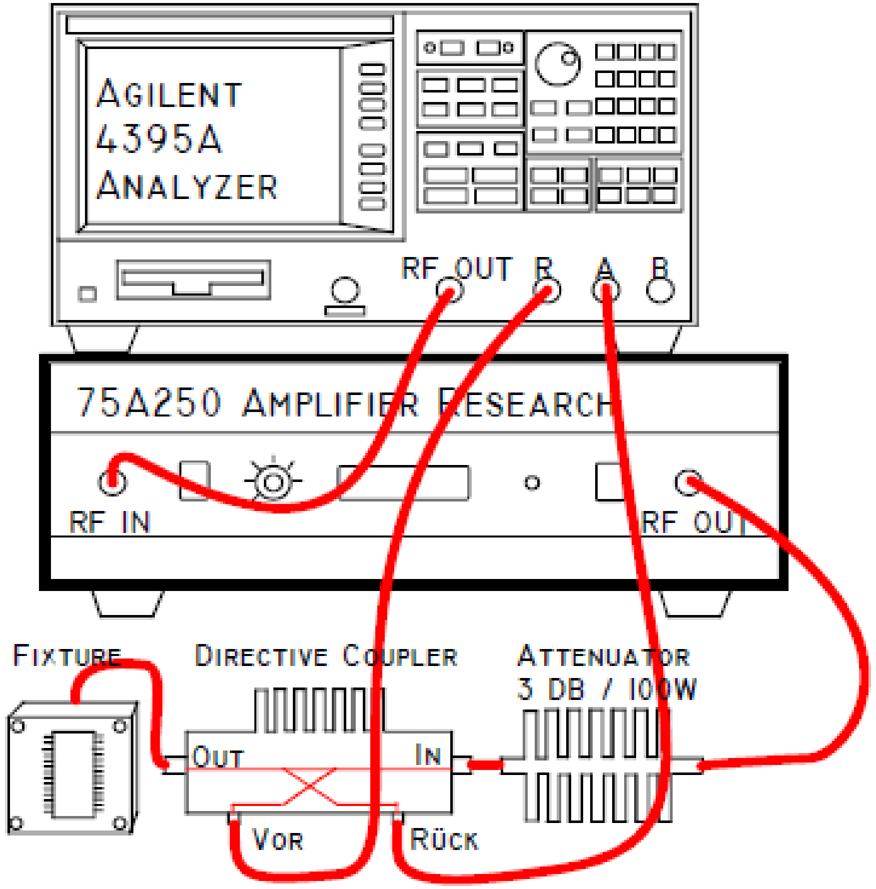




☞ Voltage @ 13,56 MHz in Matching NW is 2,5 $V_{(RMS)}$ = 7,08 $V_{(pp)}$

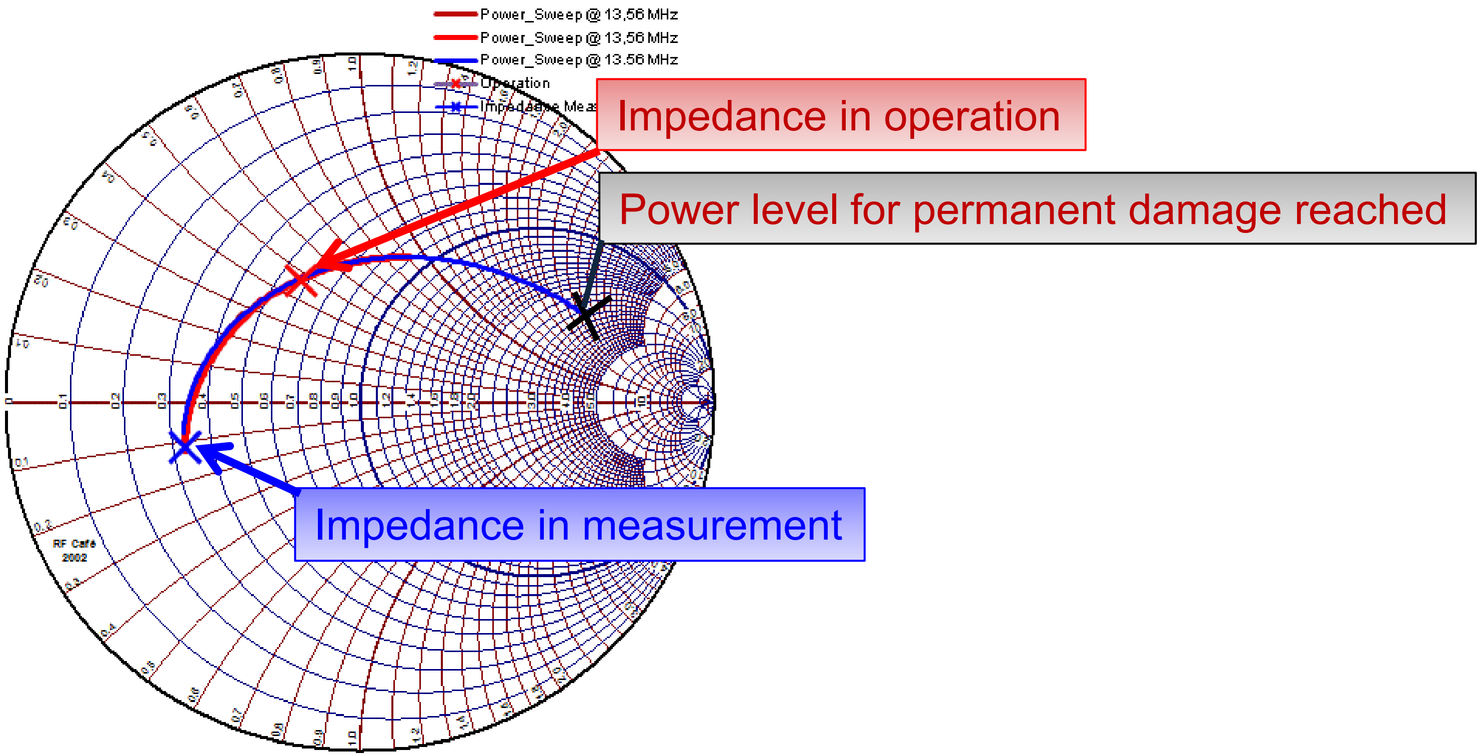
Measurement of the matching impedance
Smith Chart frequency sweep for power steps

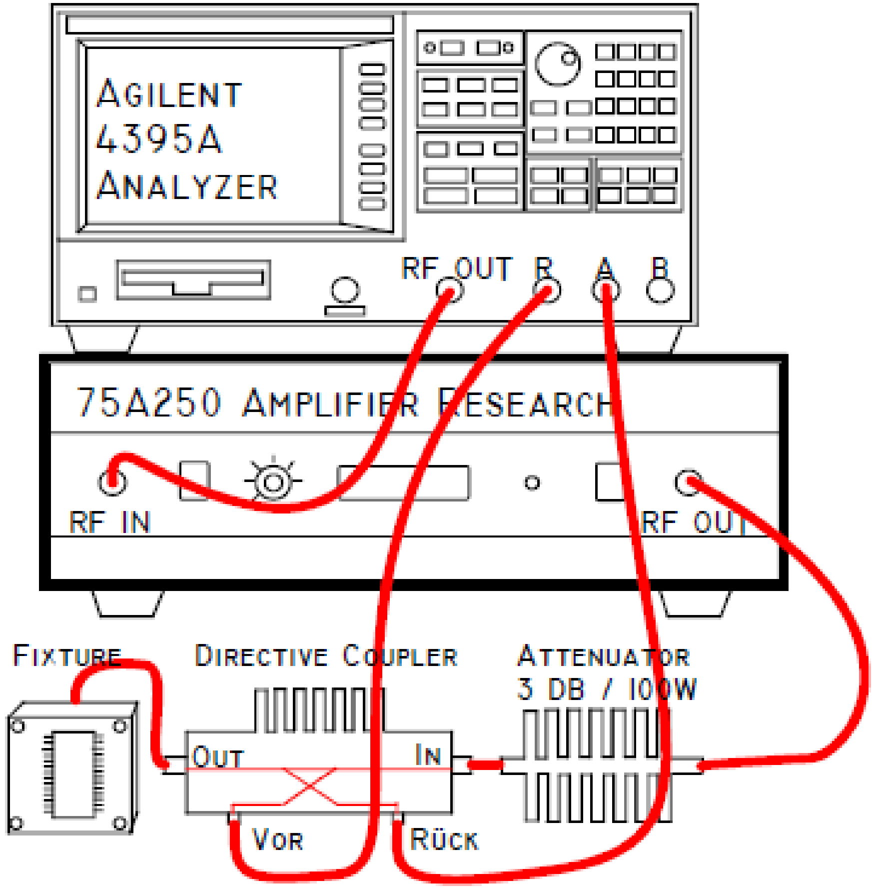




Measurement of the matching impedance

Smith Chart power sweep @ 13,56 MHz



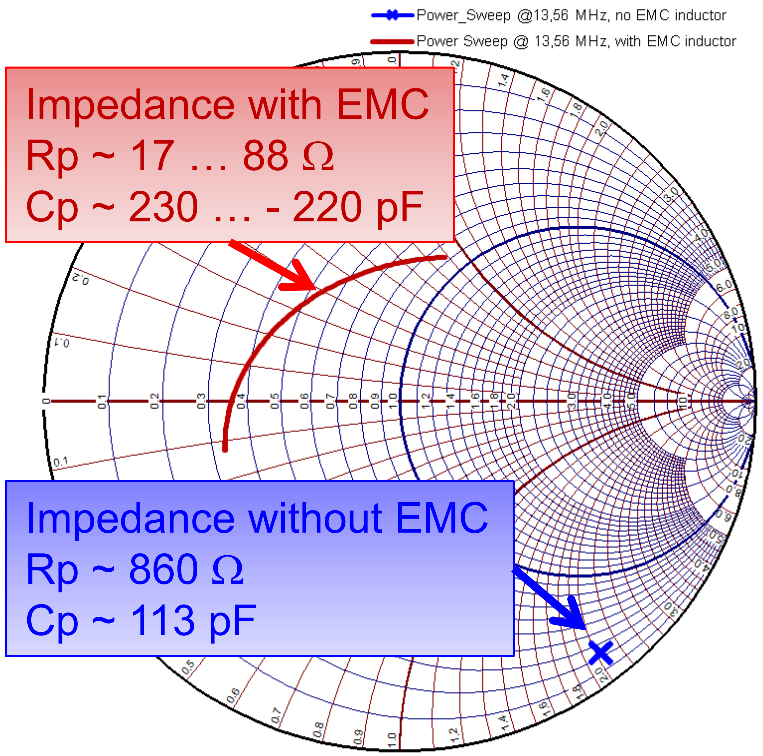


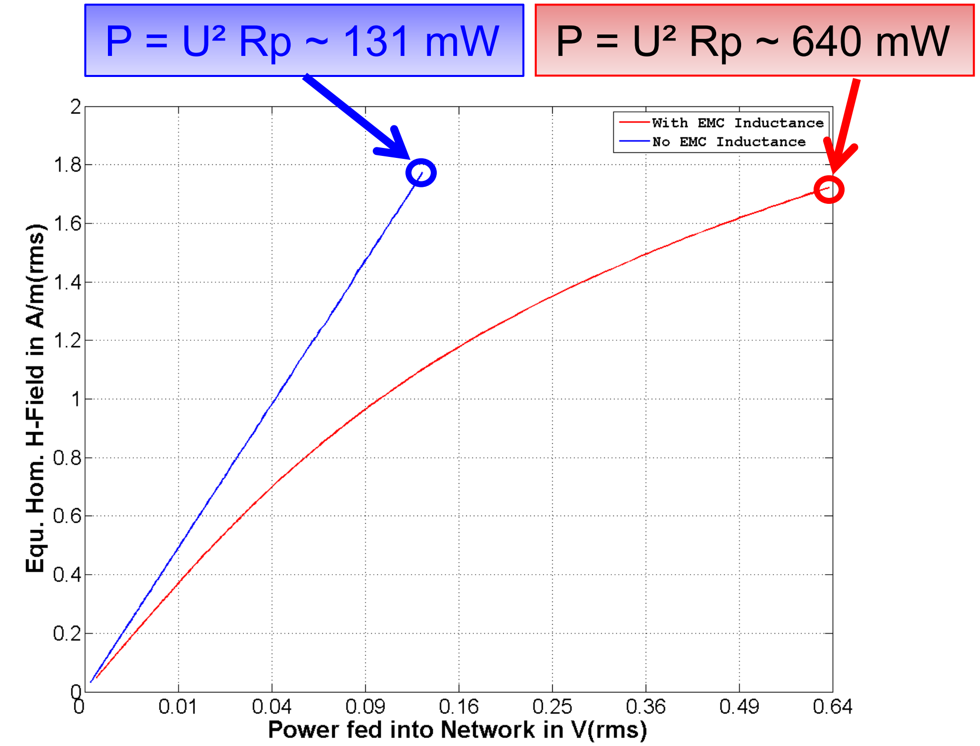
- Impedance in operation … 28,3 + j 23,9 Ω @ 3,52 $V_{(RMS)}$ = 9,95 $V_{(pp)}$
- Impedance in measurement… 16,36 – j 5,43 Ω @ 0,13 $V_{(RMS)}$ = 0,37 $V_{(pp)}$

Emitted equivalent homogenous *H*-field at CalCoil in 10 mm

Power sweep into network *with* (left) and *without* (right) EMC inductor

- Impedance, RF power & *H*-Field





☞ Almost linear increase of *H* with sqrt (RF Power)

☞ No power dependency in the remaining network, only caused by the EMC inductor

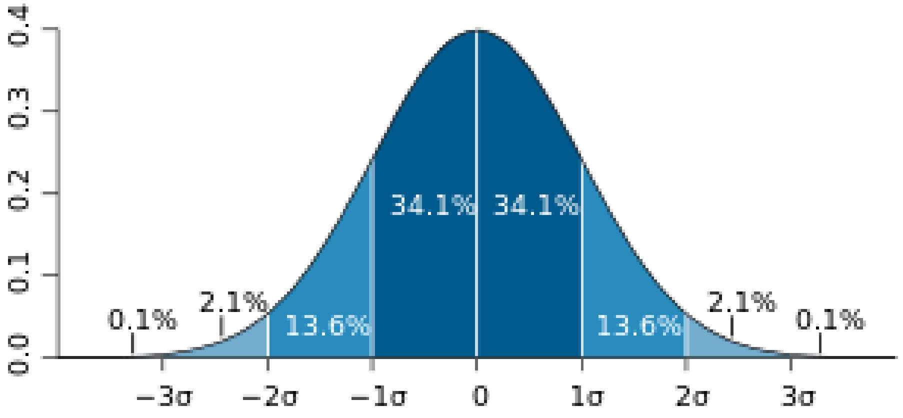
☞ Same *H*-field emitted at 131 mW RF power and at 640 mW RF power fed.

Some Statistics

Practical summary

Statistics summary

- A number of components *may have* a **Gaussian Distribution** of values.
- The Gaussian Distribution is given by the Distribution Function…



$$\Phi(z) = \frac{1}{2\pi} \cdot \int_{-\infty}^{z} (e^{-\frac{x^2}{2}})\, dx$$

- Of interest is the **Confidence Interval** of the distribution $\mu \pm z\,\sigma$
  - the probability for the value *x* to be in the Confidence Interval is $p = 2\Phi(z) - 1$
  - the confidence interval for a given probability of *x* is $z = \Phi^{-1}\left(\frac{p+1}{2}\right)$

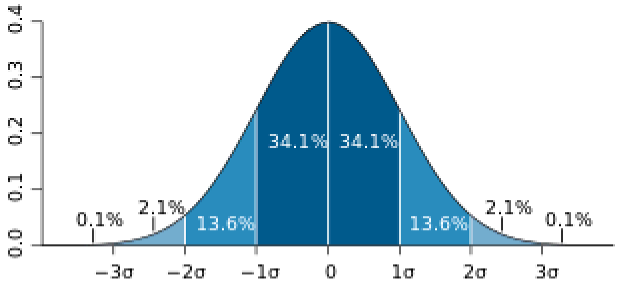
Statistics summary

- Such a Gaussian Distribution is completely determined by…

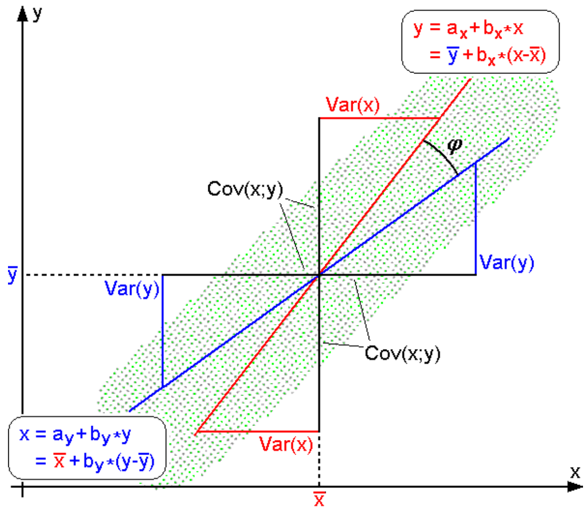
	in general	*for random sampling*
– the mean	$\mu = \int_{-\infty}^{+\infty} x\, f(x)\,dx$	$x_{AVG} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \ldots + x_n}{n}$
– the variance	$\sigma(x)^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,dx$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - x_{AVG})^2$

- The **Standard Deviation** $\sigma(x)$ is the square root of the **variance**.
- Certain **Confidence Intervals** contain samples with a certain probability…

$$\mu \pm 1\sigma \rightarrow 68{,}3\,\%$$
$$\mu \pm 3\sigma \rightarrow 99{,}7\,\%$$
$$\mu \pm 6\sigma \rightarrow 99{,}999999803\,\% \rightarrow 2\ ppm\ outside$$



Correlation



- The (Pearson product-moment) correlation coefficient r_{xy} is a linear measure for the correlation of two random variables *x*, *y*.
- *r* is in the range of -1 … 0 … +1

Definition in general

$$\rho_{xy} = \frac{\mathrm{Cov}(x, y)}{\sigma(x) \cdot \sigma(x)}$$

for random sampling

$$r_{xy} = \frac{\sum_{i=1}^{n} (x_i - x_{AVG})(y_i - y_{AVG})}{\sqrt{\sum_{i=1}^{n} (x_i - x_{AVG})^2 \cdot \sum_{i=1}^{n} (y_i - y_{AVG})^2}}$$

- For $r_{xy} = 1$ we can calculate $y = ax + b$
- Covariance is defined by…

$$Cov(x, y) = E\left[(x - \mu_x)(y - \mu_y)\right]$$

$$Cov(x, y) = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{2} (x_i - x_j)(y_i - y_j)^T$$

Adding up Gaussian Distributions

- For two Gaussian distributions for x and y, the means can be added

$$\mu_{x+y} = \mu_x + \mu_y$$

- Standard deviations cannot just be added up – variances are added!
- For **uncorrelated distributions**, the resulting variance is given by…

$$\sigma_{x+y}{}^2 = \sigma_x{}^2 + \sigma_y{}^2 \quad iff \quad r_{xy} \equiv 0$$

- This is indeed a specific case of the sum of **correlated distributions**

$$\sigma_{x+y}{}^2 = \sigma_x{}^2 + \sigma_y{}^2 \pm 2 r_{xy} \sigma_x \sigma_y \quad for \quad r_{xy} \neq 0$$

- For more than two random variables, extensions can be made, using the **covariance matrix**.

Thank you for your Audience!

Please feel free to ask questions...

Hinweise – Notizen

Questions for self-evaluation

- Remember the two dedicated loop antenna orientations: Coaxial and coplanar orientation. Talk about them, explain, how the *H*-field (and *E*-field) decrease with distance to the emitting antenna, for near-field and far-field.
- Why can we say, in the physical near-field mainly the *H*-field is relevant? Where does electromagnetic wave transmission actually start? What does "near-field" actually mean, and in which distance to the current-carrying conductor is it located?
- Which two models can we usually use, to estimate the H-field strength in distance to a loop antenna? Which pro´s and con´s does each of the methods have?
- Why do we use resonance circuits in contactless communication? What is the resonance frequency, what is the Q-factor, and how can we calculate it?
- What is the difference between electrical network elements and components? Besides the main electrical property, which parasitic properties are there, and which dependencies can exist under real-life operating conditions?
- How can we basically measure, or verify, if a component has the desired properties?

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