

# **05 HF Loop Antennas**

5<sup>th</sup> unit in course 451.417, RFID Systems, TU Graz

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## What is a loop antenna?

- The loop antenna is a distributed component with inductance (L) as main element and capacitance (C) and resistance (R) as parasitic network elements.
- For simulation it must be represented by an equivalent circuit network of lumped elements. Over a wide frequency range this can be a loose coupled reactive ladder network of resonance circuits - it has several resonances in frequency domain.
- At 13,56 MHz carrier frequency we use the fundamental (lowest) resonance. So we can simplify the equivalent circuit e.g. to a parallel resonance circuit (since losses are mainly determined by chip current consumption in Proximity Systems).
  - Note: This is a narrow-band approximation only!







## **13,56 MHz loop antenna appearence**

• NFC and Reader loop antenna



• Card & Label loop antenna



# **Smartcard & Label antennas**

Overview

#### The world of Smartcards

- ISO/IEC14443......The Contactless Proximity Air Interface for person-related applications was standardized 2 decades ago.
- Applications in Government (e-Passports, driver license, health card...), Payment (Contactless Credit Cards), Public Transport (Ticketing), Secure Access Control, etc. are successfully deployed.
- The battery-less, field-proven secure chip technology did migrate into objects e.g. SD-Cards, watches, USB-Sticks, which require small antennas. This requires more accurate characterization and production tolerance consideration.

#### Related ISO/IEC Standards

- 7810.....Card geometry (e.g. ID-1 format) and physical properties
- 7811-3/-3...Embossing (letters raised in relief)
- 7811.....magnetic stripe cards
- 7812.....optical character recognition cards
- 7813.....bank cards
- 7816.....contact cards with ICs
- 10373.....test methods



Card geometry specifications.

#### Form factors of cards

• Geometry and physical properties of cards are specified in ISO/IEC7810. 3 card formats are differentiated:



Radius of edges: 3.2 mm

Format	Length (mm)	Width (mm)	Thickness (mr
ID-1	85,60	53.98	0,76
ID-2	105,00	74,00	0,76
ID-3	125,00	88,00	0,76

#### Loop antenna geometry classes 1 ... 6 summary from ISO/IEC14443



#### **Material properties of cards**

- Card material and construction
  - Cards can consist of massive (bulk) material, of laminated sheets, or can be bonded.
  - Substrate materials: PVC, PVCA, Polyester, Polyethlyene (PE), Polycarbonate, etc.
- Ambient conditions
- Temperature range- 35 °C .... + 50 °C- Humidity5 ... 95 %- Opacityoptical density > 1,5- Warpingfor flat cards not more than 1,5 mm (this includes 0,76 mm thickness)
- Integrity

Stacked cards out of production quality control should not have any of these properties

- De-lamination (separation of individual sheets) tested with 6 N/cm<sup>2</sup> force
- De-coloration or change of color printing
- Modification of surface structure
- Deformation
- Further aspects, e.g. flame-resistance, toxicity, chemical resistance referenced to other standards. Lifetime is not specified.

#### **Construction of a transponder card**

- A special challenge for contactless transponder cards is the low profile of only 0,76 mm thickness. So chip, package and antenna must be built very flat.
- In the card production process, the pre-lam is processed first. Typically it consists of 2 inlet foils, one is embedding the antenna, and the other one has punched out holes in the chip module size, to embed the transponder. This is shown in below sketch for an embedded wire antenna:

- The prelam is stacked with two cover sheets on top and bottom, all laminated to the end-lam (White Card).
- Contactless chip cards are fabricated in sheets of e.g. 24, 36 or 48 panels.



#### Antenna and assembly technology overview



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#### **Production Technologies: Etching**



- **Process:** Standard flexible PCB fabrication process. Conductor is etched out. Good design rules (0,1 mm track width and track gap, typically)
- Material: Copper, Alu (lower costs)
  - thickness: 35  $\mu$ m, 20  $\mu$ m, 16  $\mu$ m, 8  $\mu$ m, ...
- Assembly: Copper allows welding, crimping, soldering and conductive gluing.
   Conductive gluing can be problematic for Alu because of corrosion, soldering is not possible.
- Performance:
  - Parasitic capacitance (mainly bridge cap) is relevant. Has to be considered in loop antenna design, may have high production variation. Try to keep cap value low.
  - Coils of high Q-factor, lowest (best) tolerances (if low parasitic cap)
- Applications: Reader antennas, Vicinity Label antennas (accurate resonance frequency...)

#### **Production Technologies: Embedded Wire**



- **Process:** Wire heated by ultrasound is melted into card substrate.
- Material: Copper wire
  - HF: 112  $\mu m,$  80  $\mu m$
  - LF: 50  $\mu m,$  30  $\mu m,$  20  $\mu m$
- Assembly: All processes possible, welding preferred
- Performance:
  - Low parasitic capacitance by nature (~ 1...3 pF)
  - Coils of high Q-factor, although tolerances are higher than for etching, as the coil shape can be changed during lamination.
- Application: Typical contactless personal card.



#### **Production Technologies: Electroplating**



- **Process:** First, a very thin conductive seed-layer is sputtered on the substrate. Then several µm of copper are deposited in a galvanic process.
- Material: Copper
- Assembly: Copper allows welding, crimping, soldering and conductive gluing
- Performance:
  - Coils of medium / high Q-factor, higher tolerances due to variable thickness over production and fiber-structures on the coil trace border.
- Application: E.g. contactless personal card.



#### **Production Technologies: Electroplating**



- **Process:** Conductive ink or paste is used to print the conductor,
- Material: E.g. silver ink, carbon ink, polymer paste
- Assembly: Mostly conductive gluing (pressure dependent, tolerances) or crimping
- Performance:
  - High resistance (e.g. 250  $\Omega$  before lamination, 20  $\Omega$  after lamination)  $\rightarrow$  low Q
- Applications: (Environmentally friendly) alternative for contactless personal cards

#### **Assembly: Crimping**

The next important processing step is the connection of the antenna and the transponder module. Various antenna conductor materials and concepts require appropriate connection technologies. The electrical properties of the connection also have influence on the function (or lifetime) of the transponder, so the technology should be chosen with care.

- Crimping: (Cut-Clamp-Technology)
  - The large sized, flat metal connection pads of a chip module are pierced through with a sharp tool, so that spiky crowns are formed. Consequently, the chip module is pressed against the antenna substrate foil, while the metal splitters are pierced through the foil and build a contact to the antenna conductor. The splitters are then flattened with a stamp tool, so that we find a durable mechanical and electrical connection after the lamination process.



#### **Assembly: Soldering**

- Soldering: (Reflow-Technology)
  - To avoid shorts between the antenna windings, the area for the contact pads is coated with lead-stop varnish in a first step, and only the contact points are kept accessible. On these contact points, the appropriate amount of lead paste is dispensed. After the chip module is placed in the punched-out hole of the substrate foil and so is mechanically fixed, the contact pads are heated up with a solder stamp tool (or infrared light), so that the solder paste melts and a durable connection is given.



## **Assembly: Welding, Gluing**

#### • Welding:

- Contact welding: The metal contact of the chip module, and the antenna pad are contacted with metal needle-stamps, heated up by current and connected (melted) together.
- Supersonic welding: Energy is transmitted by Sonotrodes via ultrasound to the contact points, which are heated up and melt to the other conductor.
- Gluing:
  - Conductive glue (e.g. certain epoxy) are used for mechanical fixing and electrical connection between chip and antenna conductor.

# Loop antenna design

Estimation of Equivalent Circuit Element values using an analytical formula approach

#### Way of work, procedure

- Starting point for a new antenna design are normally desired values for the equivalent circuit representation (*L*, *C*, *R*). With frame conditions (antenna size, fabrication process, manufacturer design rules) a basic design can be made, and parameters *L*, *C*, *R* can be estimated based on previous measurements, or analytically, or by FEM modeling. Especially for card antennas on soft substrate, the preferred method is to use analytical models with one parameter to fit to the process (e.g. geometry shrink).
- In a fist step, several antenna prototypes are produced, with parameter variation (e.g. number of turns) in a matrix. These are then measured, and a fitting of the model to the process is done.
- In a second step, antennas can be fabricated with properties fitting very well to the desired values.
- For complex geometries or difficult environment (ferrite materials, metal close by, e.g. for NFC interfaces in devices), a FEM based design approach is more appropriate.

#### Loop antenna design – inductance

• In the first step, antenna geometry and material properties are defined. (Edge radius is neglected for estimation of inductance).

#### Rectangular loop antenna geometry

- *a*<sub>0</sub> (maximum) length in mm, *b*<sub>0</sub> (maximum) width in mm,
- *w* conductor track width in mm,
- *g* track gap in mm,
- t track thickness in mm
- N number of turns



#### Loop antenna design – inductance

- The multi-turn loop of rectangular cross-section is recalculated to one average loop of circular cross-section:
- equivalent conductor diameter (is the diameter for wire)

$$d = 2\sqrt{\frac{t \cdot w}{\pi}}$$

• average track length

$$a_{avg} = \frac{a_0 + [a_0 - 2N \cdot (g + w)]}{2}$$

• average track width

$$b_{avg} = \frac{b_0 + [b_0 - 2N \cdot (g + w)]}{2}$$

*E* is the fitting parameter of the model, it depends on edges, radius, etc. Typical value is 1,65 (range is 1,6 .... 1,9).

• Inductance of this average loop is constituted of **mutual** 

inductance (for parallel conductor parts)....

$$M_{1} = \frac{\mu_{0}}{2\pi} \left[ a \cdot \ln \left[ \frac{2 \cdot a \cdot b}{d \cdot \left(a + \sqrt{a^{2} + b^{2}}\right)} \right] - 2b + \sqrt{a^{2} + b^{2}} \right]$$
$$M_{2} = \frac{\mu_{0}}{2\pi} \left[ b \cdot \ln \left[ \frac{2 \cdot a \cdot b}{d \cdot \left(b + \sqrt{a^{2} + b^{2}}\right)} \right] - 2a + \sqrt{a^{2} + b^{2}} \right]$$

• ...and **self-inductance** for straight parts...

$$L_1 = \frac{\mu_0 \cdot a}{16\pi} \qquad \qquad L_2 = \frac{\mu_0 \cdot b}{16\pi}$$

 Finally, all parts are summarized and the number of turns is taken into account, to estimate inductance (in μH).

$$L_{A} = (2M_{1} + 2M_{2} + 2L_{1} + 2L_{2}) \cdot N^{E}$$

#### **Example – Inductance**



- The main parameter for loop antenna design is inductance. For an air coil, this can be estimated from geometry.
  - Geometry of a several turn loop antenna is re-calculated to one average loop (length x width).
  - A rectangular conductor cross-section is approximated by a circular cross section of equal area.

 $d = 2\sqrt{\frac{t \cdot w}{\pi}}$ 

- Between parallel current lines, we consider mutual inductance from geometry ...

$$M_1 = \frac{\mu_0}{2\pi} \left[ a \cdot \ln \left[ \frac{2 \cdot a \cdot b}{d \cdot \left(a + \sqrt{a^2 + b^2}\right)} \right] - 2b + \sqrt{a^2 + b^2} \right]$$

– …and self-inductance …

$$L_1 = \frac{\mu_0 \cdot a}{16\pi}$$

 ...then we can add up all terms and take into account the number of antenna turns *N* to the power of *E* (close to 2).

$$L_{A} = (2M_{1} + 2M_{2} + 2L_{1} + 2L_{2}) \cdot N^{E}$$



Antenna geon	netry (mm)	Electrical parameters			
outline	79 x 49	Inductance	4.75 μΗ		
track width	0.7	Capacitance	3.93 pF		
track gap	0.4	DC-Resistance	2.55 Ω		
thickness	0.018	$R_A$ (@13.56)	50 k		
turns	6	Q-factor	~ 124		

#### Loop antenna design – Resistance

- We fresh platinum
   X DBS

   V(s0 playWave
   v(s0 playWave
   v(s) playWave

   ↓119
   1101
   1234
   0000

   © 00 = 007/00
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   V/SA
- Antenna losses are the result of conductor DC-resistance, and AC-losses due to skin and proximity effect. Depending on substrate material, additional (e.g. dielectric) losses may also be significant.
  - Serial DC-resistance for the planar spiral antenna can be calculated from the conductor track to

$$R_{DC} = \frac{2N(a_0 + b_0) - 2(N - 1)(w + g)}{\sigma \cdot t \cdot w}$$

- In addition, skin and proximity effect losses may have an impact, but are difficult to model, especially for rectangular conductor cross-sections, and in the magnetic field influence of neighbour turns.
- The equivalent parallel antenna resistance  $R_A$  can be calculated for one frequency (typ. carrier at 13,56 MHz)  $(2\pi f L_A)^2$

$$R_A = \frac{\left(2\,\pi\,f\,L_A\right)^2}{R_{SERIAL}}$$

- If the optimum achievable chip performance should not be significantly degraded,  $R_A > 10 R_{CHIP}$  should apply. This also means, the transponder operational Q-factor is mainly determined by the chip,  $R_T \sim R_{CHIP}$ .



Antenna geon	netry (mm)	Electrical parameters		
outline	78.6 x 23	Inductance	1.64 μH	
track width	0.8	Capacitance	3.74 pF	
track gap	0.2	DC-Resistance	0.65 Ω	
thickness	0.035	$R_A$ (@13.56)	22 k	
turns	4	Q-factor	~ 155	

#### Loop antenna design – Capacitance

- Parasitic capacitance for planar loop coils consists of up to 3 contributions:
  - Bridge capacitance
    - The conductor bridge between end of inner turns and end of outer turn builds up a plate capacitor. (may add 10 pF)

- Turn capacitance

- Considers area and distance between (n-1) turns, and voltage phase-shift (may add 2 pF)
- Electrical length
  - Resonance condition if the electrical track length is equal to half the wavelength of a resonance frequency (may be < 1 pF)</li>





## Loop antenna measurement

Overview

#### Antenna equivalent parameter measurement

- The complex impedance of a loop antenna can be measured with an Impedance or Network Analyzer over frequency.
- An equivalent circuit (EQC) consisting of lumped elements is extracted from such a trace.
- Most simple this can be a parallel resonant circuit, consisting on Inductance L<sub>A</sub>, Resistance R<sub>A</sub> and Capacitance C<sub>A</sub>. This is most accurate for the carrier frequency.

self-resonance

frequency



- $C_A$  is calculated from self-resonance frequency and  $L_A$  $f_S = \frac{1}{2\pi \sqrt{L_A C_A}} \rightarrow C_A = \frac{1}{(2\pi f_S)^2 L_A}$
- *R<sub>A</sub>* either can be measured as *Re{Z}* at 13,56 MHz, or can be calculated from





inductance

0-

inductive behaviour

coil inductance

best point to read out

## Measurement with Agilent 4395A (1)

- Switch on Instrument
  - Preset
- Use Impedance-Analyzer Mode
  - Meas => Analyzer Type => Impedance Analyzer
- Choice of frequency range 1 100 MHz
  - Start => 1 => MHz
  - Stop => 100 => MHz
  - Sweep => number of points => 801 => x1
- Set to Inductance Measurement
  - Meas => More => Ser (Ls)
- Calibrate Instrument
  - Cal => Cal Kit => 3,5 mm => Return
  - Cal => Calibrate Menu
  - connect calibration kit to test fixture and check good connection,
  - Open, Short, Load, Done
  - Leave 50-Ohm reference connected and check successful calibration using...
  - Scale Ref => Autoscale





## **Measurement with Agilent 4395A (2)**

- Compensate Fixture (measurement adaptor)
  - Cal => Fixture Compen => Compen Menu
  - Connect a SMA-connector with open contacts to fixture
  - => Open
  - solder open contacts to get a short
  - => Short, Done
  - Check: Flat trace over frequency
- Actual measurement of antenna parameters
  - Meas => Ser(Ls)
  - Marker => 1 => MHz
  - Read out value
  - Meas => More => Ser(Rs)
  - Read out value
  - Meas => More => Ser(Ls)
  - Manually set the marker to the zero crossing, Read out the (resonance) frequency
  - Meas => More => Prl(Rp)
  - Read out Rp at fres



#### **Measurement with Agilent 4395A (3)**



#### An alternative *contactless* measurement

• Procedure:

- Two capacitors of well-known capacitance are consecutively connected to the loop coil (DUT). C1, C2
- Measure resonance frequency for both cases.
- Unknown loop antenna inductance can be calculated from the resonance frequency equations:  $fres_1 = \frac{1}{2\pi\sqrt{Ls\cdot(C_1+C_p)}}$

 $Ls = \frac{\left(\frac{1}{2\pi fres_{1}}\right)^{2} - \left(\frac{1}{2\pi fres_{2}}\right)^{2}}{|C_{1} - C_{2}|}$ 

- $fres_2 = \frac{1}{2\pi\sqrt{Ls\cdot(C_2+C_P)}}$
- The parasitic capacitance applies for both resonance frequencies, in parallel to the well-known capacitors.
- So it can be calculated according to...

$$Cp = \frac{\frac{1}{\left(2\pi fres_{1}\right)^{2}}}{L_{s}} - C_{1}$$

#### **Example measurement series – embedded wire antennas**



- Parameterized Class 2 loop antenna matrix run
- Embedded wire antenna fabrication

	Embedded Wire rectangular Antennas								
S	outline	mm	78,6 x 23	78.6 x 23	78.6 x 23				
ion	wire diam.	mm	0,112	0,112	0,112	0,112	0,08	0,112	0,112
su	wire pitch	μm	0,2	0,2	0,2	0,2	0,2	0,4	0,4
me									
Di									
	turns		3	4	5	6	5*	6*	6,8*
ed	Ls @ 1MHz	μH	1,68	2,79	4,14	5,73	4,32	4,79	6,02
nre	Rs Q 1 MHz	Ohm	1,24	1,65	2,08	2,47	3,72	2,19	2,52
eas	fres	MHz	70,08	55,13	45,45	38,89	46,86	51,00	43,35
Ĕ	Rp	kOhm	42,62	63,30	88,70	100,81	72,10	200,00	150,00
ted									
ılat	Ср	pF	3,07	2,99	2,96	2,92	2,67	2,03	2,24
IICT	Rs	Ohm	1,45	2,09	2,85	3,87	4,73	2,62	3,50
Ca	Q		98,62	113,59	123,93	126,28	77,80	155,80	146,50

#### **Example measurement series – etched antennas**



- Parameterized Class 2 loop antenna matrix run
- Etched antenna fabrication

		Etc	hed	recta	ngul	ar Ar	ntenr	las	
S	outline	mm	78,6 x 23	78,6 x 23	78,6 x 23	78,6 x 23	80,6 x 25	78.6 x 23	78.6 x 23
ion	track width	mm	0,8	0,8	0,8	0,8	0,8	0,5	0,8
su	gap width	mm	0,2	0,2	0,2	0,2	0,2	0,2	0,2
me									
D	Cu thickness	mm	0,035	0,035	0,035	0,035	0,035	0,035	0,035
		Inlay	Inlay	Inlay	Inlay	Inlay	Inlay	Inlay	Card
	turns		3	4	5	6	7	8*	4*
ed	Ls @ 1MHz	μH	1,04	1,64	2,28	2,93	3,90	5,51	1,63
sur	Rs Q 1 MHz	Ohm	0,57	0,78	1,03	1,14	1,45	2,20	0,67
eas	fres	MHz	74,58	50,97	39,49	33,79	25,78	23,30	44,93
Ĕ	Rp	kOhm	18,78	15,84	15,00	15,10	13,60	19,10	14,08
ted									
ulai	Ср	pF	4,38	5,95	7,12	7,57	9,77	8,47	7,70
alcı	Rs	Ohm	0,59	1,42	2,50	3,75	7,34	11,00	1,42
ů	Q		118,42	98,70	77,53	66,49	45,28	42,67	97,63

# **NFC & Proximity Reader Loop Antennas**

Overview

#### Ferrite foil – functional principle

- Ferrite material can conduct the magnetic flux multiple times better than free air.
- If the application requires an NFC antenna to be very close to a metal plate, a thin ferrite foil can help to isolate the antenna from the metal.



- It is important to note, compared to an antenna in free air, the contactless performance will still be degraded but not completely blocked.
- Only a part of the magnetic flux is conducted in thin ferrite presence or absence of metal in close coupling below will detune the antenna matching!

#### Ferrite foil – when to use...?

- If the application requires an NFC antenna to be very close to a metal plate, a thin ferrite foil can help to magnetically "isolate" the antenna from the metal.
- A practical experiment did result the following:



- Air coil allows no function very close to metal plate,
- Equal function in ~ 10 mm distance,
- In free air the air coil will perform always better...

### Ferrite foil – 2 types are available

• Polymer "absorber" foil



- $\text{Re} (\mu_R) \sim 20 \dots 60,$
- mechanically very flexible,
- available also on reels (simple antenna production process)
- higher conductivity (losses)

#### • Sintered ferrite sheets



- $\text{Re} (\mu_R) \sim 100 \dots 200,$
- mechanically rather rigid,
- available in sheets (e.g. 200 x 200 mm)
- very low conductivity losses

#### NFC loop antennas on ferrite foils

- Smart mobile devices are very thin and compact, offer little extra space.
- Metal parts or other RF components may be very close to NFC antenna (e.g. antenna on battery pack)
- Ferrite Foils can conduct the magnetic flux and allow dense packaging
  - Real part of  $\mu_R$  determines how much magnetic flux can be conducted ("good" property) should be very high (e.g. 120 ... 160)
  - Imaginary part of  $\mu_R$  means HF losses ("bad" property) should be low (e.g. < 3)





#### **Example: NFC Antenna**

- Inductive Loop antenna (planar spiral coil), *H*-field antenna
- Conductor on ferrite foil, to magnetically isolate antenna from metal and electronic PCB



Antenna Geometry Data							
	Dimension	Air Coil	Coil on ferrite				
ferrite outline	mm		39 x 39				
antenna outline	mm	35 x 35	35 x 35				
track width	mm	0.4	0.4				
track gap	mm	0.4	0.4				
number of turns		4	4				
E	quivalent Circu	it Element Valu	es				
Inductance L <sub>A</sub>	μH	1.314	1.980				
Capacitance C <sub>A</sub>	pF	2.351	2.922				
Resistance R <sub>A</sub>	Ω	0.58	0.85				

#### **Temperature dependency** for NFC antenna

- Air coil (blue)
- Coil on ferrite (red)
  - Specific conductance has a significant temperature gradient
  - L on ferrite has a temperature dependency (air coil has not)
  - Matching network (with Q) will increase this impedance temperature dependency!



#### Loop antenna production tolerances

- Production tolerances require to measure an average of several parts
- Errors differentiate into offset (deterministic) and variance (random)
  - Soft substrate (e.g. Cards) can shrink during lamination process
  - Ferrite foils have typically...
    - +/- 15 % tolerance of  $\mu_R$
    - $\rightarrow$  may be +/- 7,5 % for  $L_A$
- Example: NFC antenna on ferrite foil
  - Matching network for Q ~ 7
  - 8 samples measured @ 25 °C
  - Avg. impedance 17,84 + j 1,78  $\Omega$





# Thank you for your Audience!

Please feel free to ask questions...

Hinweise – Notizen

#### **Questions for self-evaluation**

- Which loop antenna mass-production methods do you know? How are the antennas produced? Which electrical & other properties can be expected?
- Which methods do you know, to assemble the transponder chip? Explain pro's and con's, and the relation to loop antenna production!
- How is a contactless card fabricated? Which layers, properties, sizes can you recall? Where is the transponder layer?
- How to design a loop antenna for contactless communication? What are main properties, and which structure may an equivalent circuit have? Which effects are important?
- How to measure properties of a loop antenna for contactless communication? Which properties are of interest?
- Can you explain the differences in loop antenna design for NFC devices and readers, and for contactless transponder cards?
- Ferrite for which cases may it be useful? Which properties does it have? In which form is it available?

#### Calculate loop antenna $C_P$ from contactless measurement

• Starting point: 
$$C_{1}, C_{2}$$
  
 $f_{R,1} = \frac{1}{2\pi\sqrt{L_{s} \cdot (C_{1} + C_{p})}}$   
 $(2\pi f_{R,1})^{2} = \frac{1}{L_{s} \cdot (C_{1} + C_{p})}$   
 $\left(2\pi f_{R,1}\right)^{2} = L_{s} \left(C_{1} + C_{p}\right) = L_{s} C_{1} + L_{s} C_{p}$   
 $\left(\frac{1}{2\pi f_{R,1}}\right)^{2} = L_{s} \left(C_{1} + C_{p}\right) = L_{s} C_{1} + L_{s} C_{p}$   
 $\left(\frac{1}{2\pi f_{R,2}}\right)^{2} = L_{s} \left(C_{2} + C_{p}\right) = L_{s} C_{2} + L_{s} C_{p}$ 

$$\left(\frac{1}{2\pi f_{R,1}}\right)^2 - \left(\frac{1}{2\pi f_{R,2}}\right)^2 = L_S C_1 + L_S C_P - L_S C_2 - L_S C_P$$

so we get 
$$L_S = \frac{\left(\frac{1}{2\pi f_{R,1}}\right)^2 - \left(\frac{1}{2\pi f_{R,2}}\right)^2}{|C_1 - C_2|}$$
 and  $C_P = \frac{\frac{1}{(2\pi f_{R,1})^2}}{L_S} - C_1$ 

RFID Systems Lecture Course at TU Graz provided by Dr. Michael Gebhart

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